Three-Point Functions in $\mathcal{N}=4$ SYM from INTEGRABILITY

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Motivations

Solving Integrable Models

Spectrum

- Find $H|\psi_n\rangle = E_n|\psi_n\rangle$
- Spectral problem in $\mathcal{N}=4$ SYM

  Find scaling dimensions $\{\Delta\}$
- Non-perturbative methods :

  (1) **S-matrix** bootstrap

  (2) Asymptotic Bethe Ansatz

  (3) TBA, Y-system, QSC ...

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Correlation Functions

- Compute $\langle\psi_a|O_1\cdots O_N|\psi_b\rangle$
- Correlation functions in $\mathcal{N}=4$ SYM

  in particular, 3pt functions

  OPE coefficients $\{C_{123}\}$
- Non-perturbative methods :

  (1) **Form factor** bootstrap

  (2) Asymptotic volume corrections

  (3) Wrapping corrections

Solving Integrable Models

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better understood

Correlation Functions

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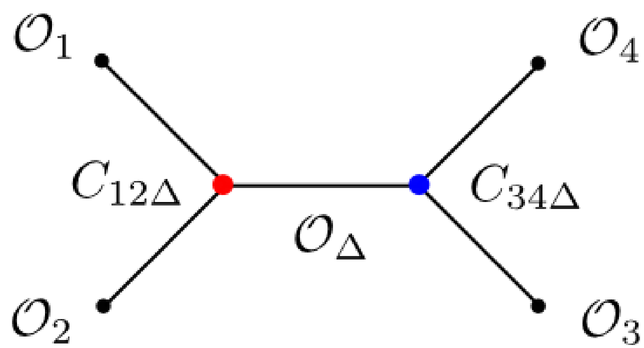
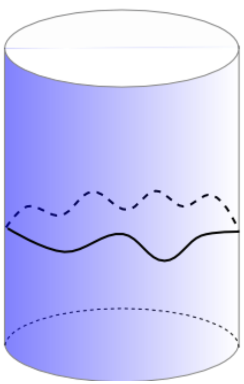
much more challenging

CFT Data & Holography

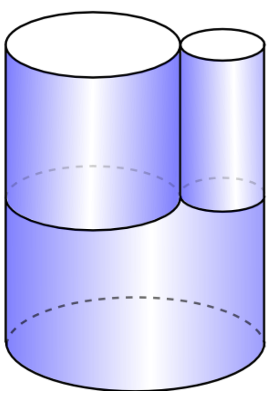
Conformal bootstrap:



AdS/CFT



2pt function



3pt function

Tree Level

[Escobedo, Gromov, Sever, Vieira, 2010-2012]
[Foda, 2012]
[Foda, Y. J. , Kostov, Serban, 2013]


Cuisine
✓Ingredients
✓Recipe
✓Cooking Skills


3Pt-Function
✓Operators
✓Gluing rules
✓Calculation

Structure constant

Conformal symmetry

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\rangle = \frac{C_{123}}{|x_{12}|^{\Delta_{12}}|x_{23}|^{\Delta_{23}}|x_{31}|^{\Delta_{31}}}$$

Structure constant

Conformal symmetry

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\rangle = \frac{C_{123}}{|x_{12}|^{\Delta_{12}}|x_{23}|^{\Delta_{23}}|x_{31}|^{\Delta_{31}}}$$

Perturbative expansion

$$N_c C_{123} = C_{123}^{(0)} + g^2\, C_{123}^{(1)} + \cdots \qquad g^2 = \frac{\lambda}{16\pi^2}$$

Tree level

One loop

Operators

- Single-trace, gauge invariant local operators

- Fundamental fields of $\mathcal{N}=4$ SYM :

  1 gauge field A_μ , 6 scalars Φ_i , 8 fermions $\psi_{a\alpha}$, $\bar{\psi}^a_{\dot{\alpha}}$

- Closed sectors, *e.g.* SU(2), SL(2), SU(1|1)…

  SU(2) sector : $\{Z, X\}$, $Z = \Phi_5 + i\Phi_6$ $X = \Phi_1 + i\Phi_2$

- From operators to spin chain states

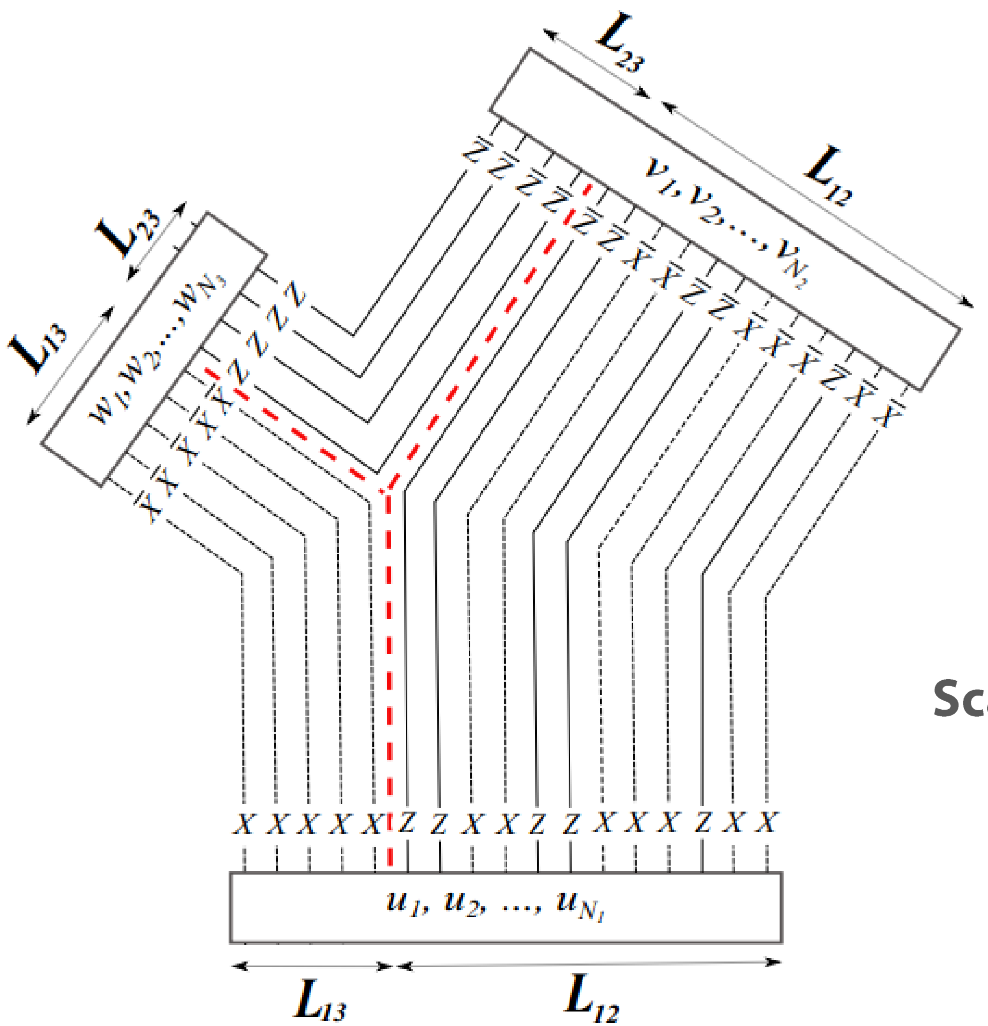
$$\mathrm{tr}\, Z X Z Z X Z \Longrightarrow |\uparrow\downarrow\uparrow\uparrow\downarrow\uparrow\rangle$$

  Operators can be constructed by **Bethe ansatz**

$$|\mathbf{u}\rangle = B(u_1)\cdots B(u_N)|\uparrow^L\rangle \qquad \{u_1, \cdots, u_N\}$$

Bethe states **Bethe roots**

Recipe



$$\langle \mathrm{tr}\, \bar{Z} \bar{X}\ \ \mathrm{tr}\, X Z \rangle = 1$$

Planar Wick Contraction

Scalar products of Bethe States

$$\langle \downarrow \uparrow | \downarrow \uparrow \rangle = 1$$

Tailoring

Cutting

$$|\mathbf{u}\rangle \to \sum_{\mathbf{u}'\cup\mathbf{u}''=\mathbf{u}} |\mathbf{u}'\rangle \otimes |\mathbf{u}''\rangle$$

$$|\mathbf{v}\rangle \to \sum_{\mathbf{v}'\cup\mathbf{v}''=\mathbf{v}} |\mathbf{v}'\rangle \otimes |\mathbf{v}''\rangle$$

$$|\mathbf{w}\rangle \to \sum_{\mathbf{w}'\cup\mathbf{w}''=\mathbf{w}} |\mathbf{w}'\rangle \otimes |\mathbf{w}''\rangle$$

Flipping

$$|\mathbf{u}'\rangle \otimes |\mathbf{u}''\rangle \to |\mathbf{u}'\rangle \otimes \langle \mathbf{u}''^*|$$

$$|\mathbf{v}'\rangle \otimes |\mathbf{v}''\rangle \to |\mathbf{v}'\rangle \otimes \langle \mathbf{v}''^*|$$

$$|\mathbf{w}'\rangle \otimes |\mathbf{w}''\rangle \to |\mathbf{w}'\rangle \otimes \langle \mathbf{w}''^*|$$

Sewing

$$C_{123}^{(0)} \sim \sum_{\text{partitions}} \frac{\langle \mathbf{u}''^*|\mathbf{v}'\rangle\langle \mathbf{v}''^*|\mathbf{w}'\rangle\langle \mathbf{w}''^*|\mathbf{u}'\rangle}{\sqrt{\langle \mathbf{u}|\mathbf{u}\rangle\langle \mathbf{v}|\mathbf{v}\rangle\langle \mathbf{w}|\mathbf{w}\rangle}}$$

Tailoring

Cutting

$$|\mathbf{u}\rangle \to \sum_{\mathbf{u}'\cup\mathbf{u}''=\mathbf{u}} |\mathbf{u}'\rangle \otimes |\mathbf{u}''\rangle$$
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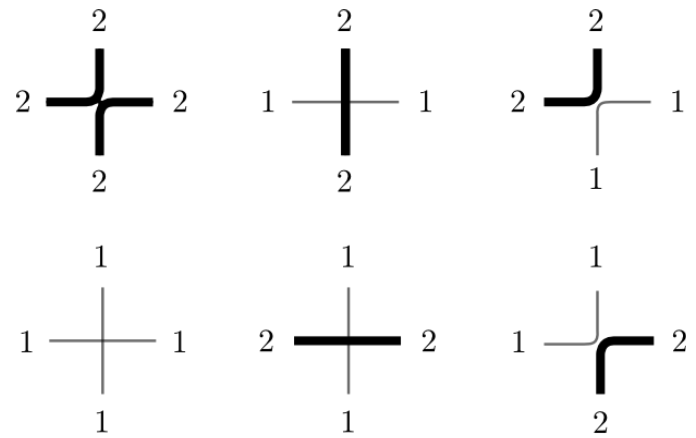
$2^{N_1+N_2+N_3}$ **terms**

Off-shell /off-shell scalar products

Freezing

XXX chain = 6 vertex model

$$R_{an}(u,\theta) = \begin{pmatrix} a(u,\theta) & 0 & 0 & 0 \\ 0 & b(u,\theta) & c(u,\theta) & 0 \\ 0 & c(u,\theta) & b(u,\theta) & 0 \\ 0 & 0 & 0 & a(u,\theta) \end{pmatrix}$$



$a(u,\theta)$ $b(u,\theta)$ $c(u,\theta)$

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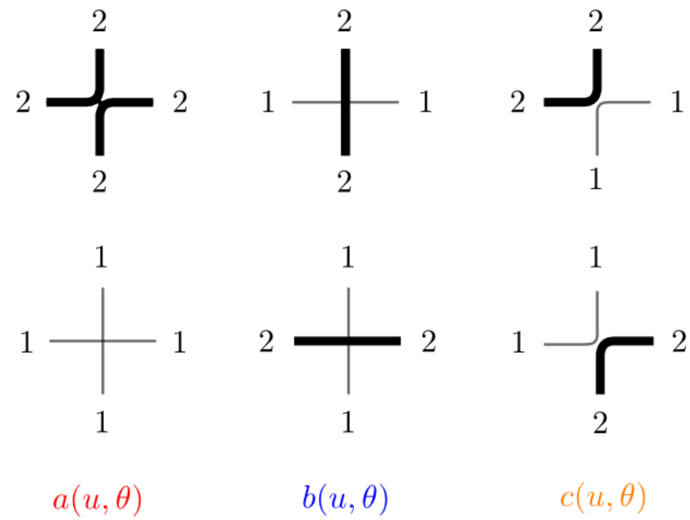
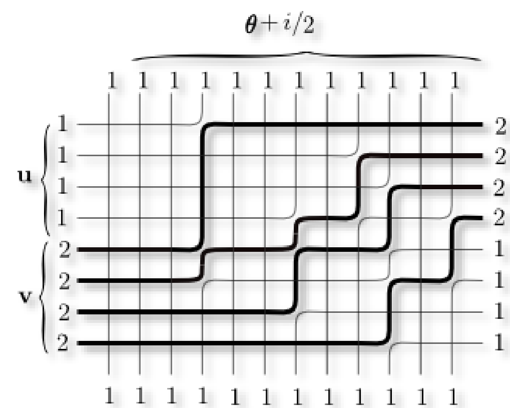


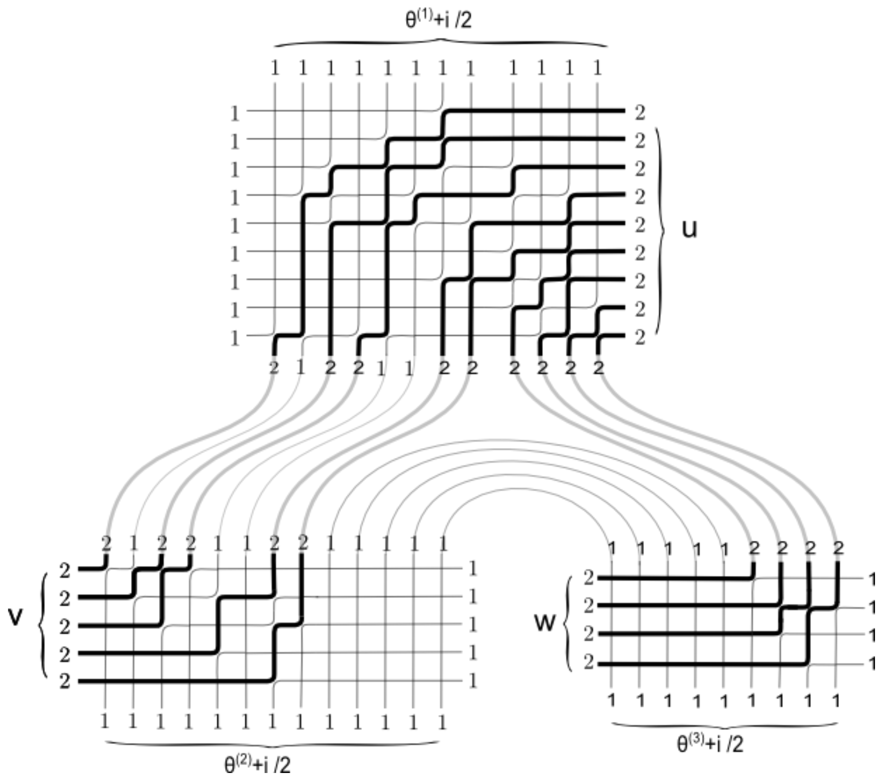
Scalar product = Partition function

$$\langle \mathbf{u}; \boldsymbol{\theta} | \mathbf{v}; \boldsymbol{\theta} \rangle$$



Freezing

The cubic vertex



Freezing

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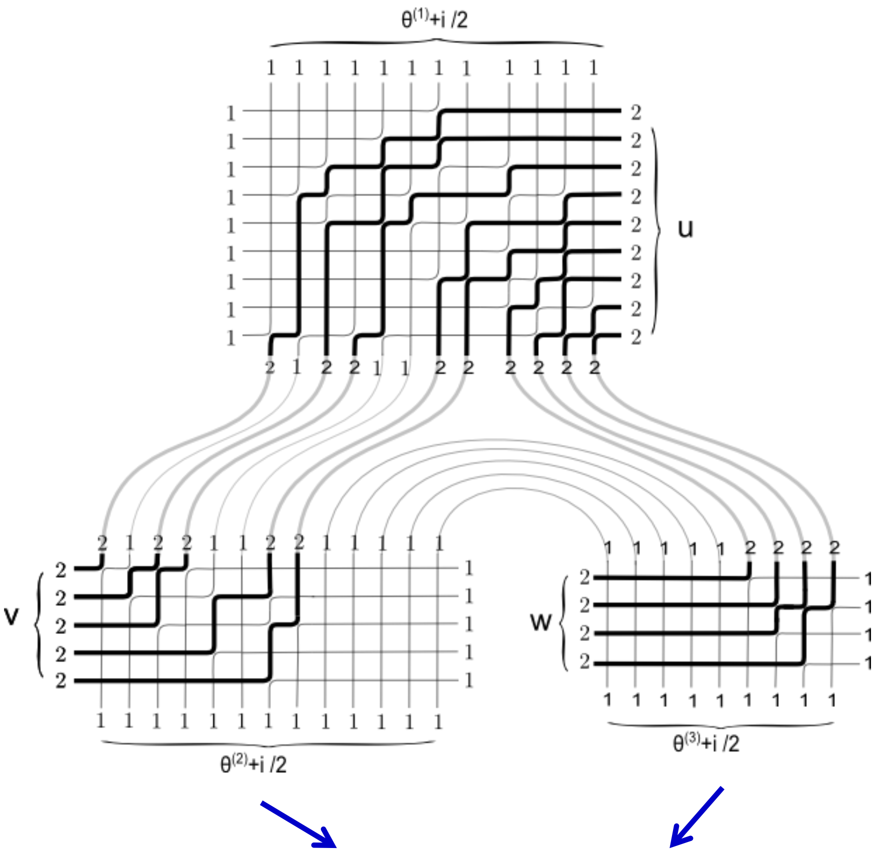


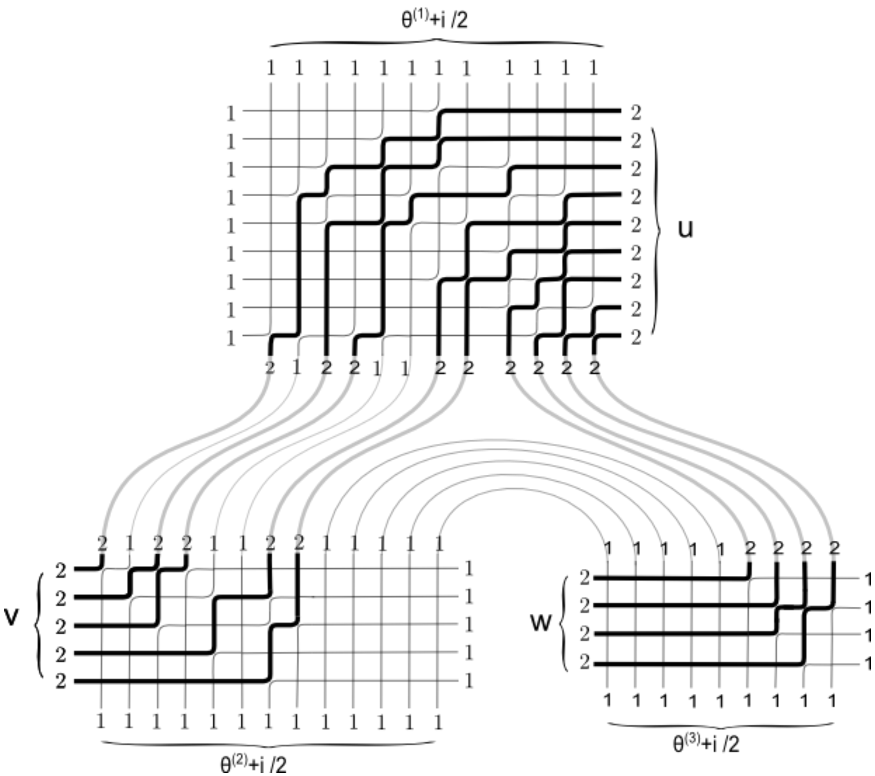
Inhomogeneities

Final result

$$C_{123}^{(0)} = \frac{\langle \mathbf{v}\cup\mathbf{z}|\mathbf{u}\rangle_{\boldsymbol{\theta}^{(1)}}\,\langle \mathbf{w}|\mathbf{z}\rangle_{\boldsymbol{\theta}^{(3)}}}{\sqrt{\langle \mathbf{u}|\mathbf{u}\rangle\langle \mathbf{v}|\mathbf{v}\rangle\langle \mathbf{w}|\mathbf{w}\rangle}}$$

Freezing

The cubic vertex



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No sum-over partition !

On-shell/off-shell scalar product
(Determinant formula)

NOTE

Works only for special configurations

Semi-classical limit

- Length of spin chain L, number of magnons $N \to \infty$
  the ratio $\alpha = N/L$ is fixed.

- Bethe roots condense into cuts

- Nice analytic results, algebraic curve description

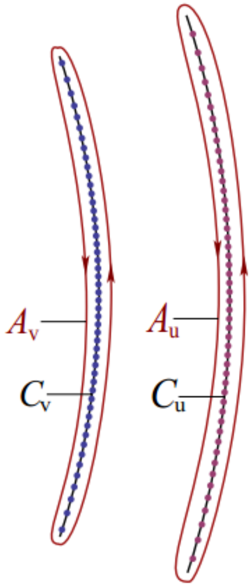
- Comparison to string theory results

- Classical statistics mechanics problem (free fermions)
  Meyer expansion, SUSY localization, …

Semi-classical limit

$$\log C_{123}^{(0)} = \oint_{A_{\mathbf{u}} \cup A_{\mathbf{v}}} \frac{\mathrm{d}u}{2\pi} \mathrm{Li}_2 \left(e^{ip_{\mathbf{u}} + ip_{\mathbf{v}} + \frac{i}{2} G_{\boldsymbol{\theta}^{(3)}}} \right)$$
$$+ \oint_{A_{\mathbf{w}}} \frac{\mathrm{d}u}{2\pi} \mathrm{Li}_2 \left(e^{ip_{\mathbf{w}} + \frac{i}{2} G_{\boldsymbol{\theta}^{(2)}} - \frac{i}{2} G_{\boldsymbol{\theta}^{(1)}}} \right)$$
$$- \oint_{A_{\mathbf{u}}} \frac{\mathrm{d}u}{4\pi} \mathrm{Li}_2 \left(e^{2ip_{\mathbf{u}}} \right)$$
$$- \oint_{A_{\mathbf{v}}} \frac{\mathrm{d}u}{4\pi} \mathrm{Li}_2 \left(e^{2ip_{\mathbf{v}}} \right)$$
$$- \oint_{A_{\mathbf{w}}} \frac{\mathrm{d}u}{4\pi} \mathrm{Li}_2 \left(e^{2ip_{\mathbf{w}}} \right)$$

Contour



[Gromov, Server, Vieira, 2011]
[Kostov, 2012]

Semi-classical limit

$$\log C_{123}^{(0)} = \oint_{A_{\mathbf{u}}\cup A_{\mathbf{v}}} \frac{du}{2\pi}\mathrm{Li}_2\left(e^{ip_{\mathbf{u}}+ip_{\mathbf{v}}+\frac{i}{2}G_{\boldsymbol{\theta}^{(3)}}}\right)$$
$$+\oint_{A_{\mathbf{w}}} \frac{du}{2\pi}\mathrm{Li}_2\left(e^{ip_{\mathbf{w}}+\frac{i}{2}G_{\boldsymbol{\theta}^{(2)}}-\frac{i}{2}G_{\boldsymbol{\theta}^{(1)}}}\right)$$
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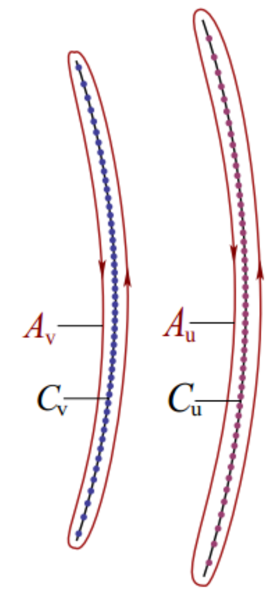


Quasi-momentum

$$\boxed{p_{\mathbf{u}}(u) = G_{\mathbf{u}}(u) - \frac{1}{2}G_{\boldsymbol{\theta}}(u)}$$

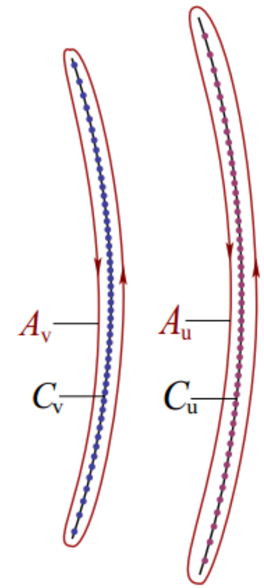
Fundamental in spectral problem

[Gromov, Server, Vieira, 2011]
[Kostov, 2012]

Semi-classical limit

$$
\begin{aligned}
\log C_{123}^{(0)} = & \oint_{A_{\mathbf{u}} \cup A_{\mathbf{v}}} \frac{\mathrm{d}u}{2\pi} \mathrm{Li}_2 \left(e^{i p_{\mathbf{u}} + i p_{\mathbf{v}} + \frac{i}{2} G_{\boldsymbol{\theta}^{(3)}}} \right) \\
& + \oint_{A_{\mathbf{w}}} \frac{\mathrm{d}u}{2\pi} \mathrm{Li}_2 \left(e^{i p_{\mathbf{w}} + \frac{i}{2} G_{\boldsymbol{\theta}^{(2)}} - \frac{i}{2} G_{\boldsymbol{\theta}^{(1)}}} \right) \\
& - \oint_{A_{\mathbf{u}}} \frac{\mathrm{d}u}{4\pi} \mathrm{Li}_2 \left(e^{2 i p_{\mathbf{u}}} \right) \\
& - \oint_{A_{\mathbf{v}}} \frac{\mathrm{d}u}{4\pi} \mathrm{Li}_2 \left(e^{2 i p_{\mathbf{v}}} \right) \\
& - \oint_{A_{\mathbf{w}}} \frac{\mathrm{d}u}{4\pi} \mathrm{Li}_2 \left(e^{2 i p_{\mathbf{w}}} \right)
\end{aligned}
$$

Contour



Quasi-momentum

$$\boxed{p_{\mathbf{u}}(u) = G_{\mathbf{u}}(u) - \frac{1}{2} G_{\boldsymbol{\theta}}(u)}$$

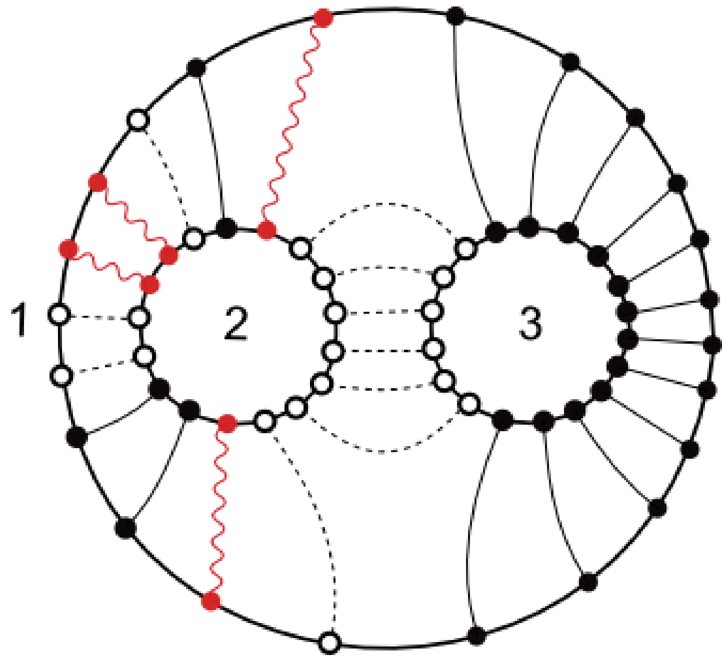
Fundamental in spectral problem

Inhomogeneities

Tree level put to zero
One loop put to BDS value

[Gromov, Server, Vieira, 2011]
[Kostov, 2012]

Comment for other sectors



Higher rank SU(3) sector

$$(X, Y, Z)$$

More kinds of 3pt functions

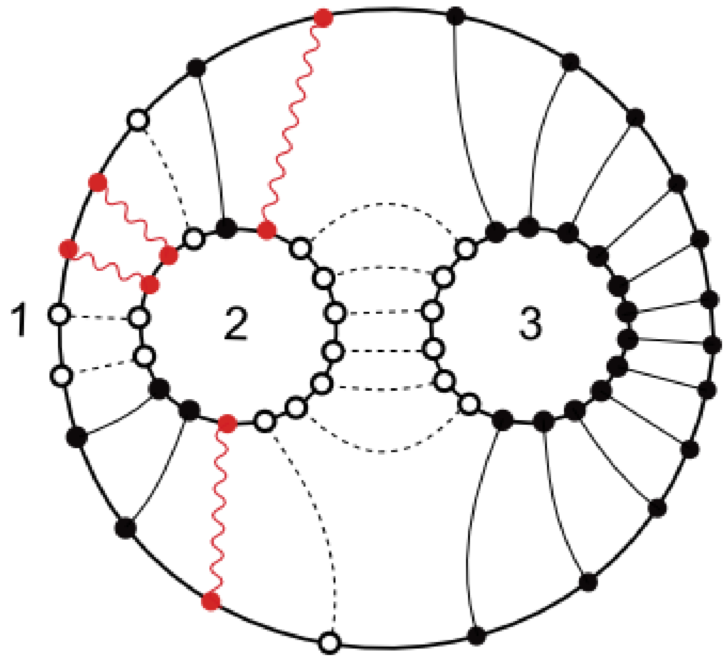
Eigenstates constructed by nested BA

Involved formulas for scalar products

[Vieira and Wang, 2013]

[Caetano and Fleury, 2014]

Comment for other sectors



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Non-compact SL(2) sector $(\ \mathcal{D}, Z\ )$ **:**

Non-compact spin chains

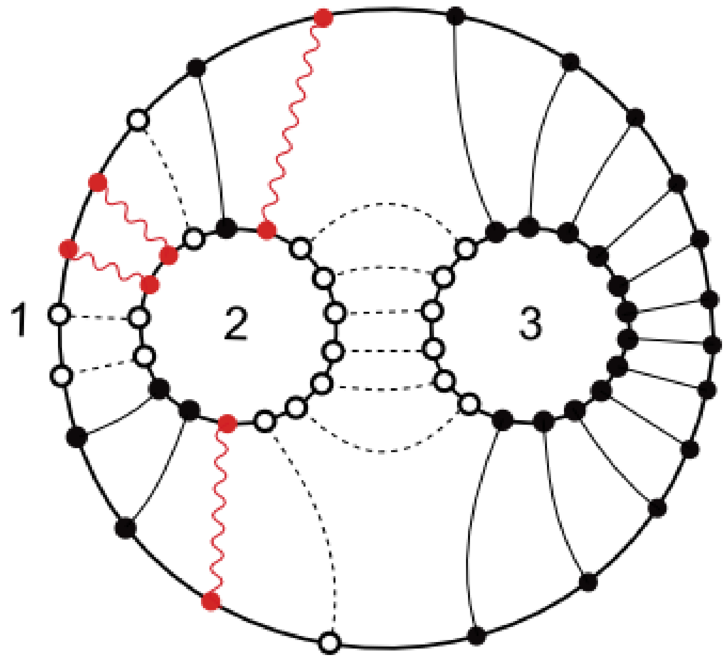
Tensor structures

Related to 4pt functions

[Vieira and Wang, 2013]

[Caetano and Fleury, 2014]

Comment for other sectors



Higher rank SU(3) sector

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Non-compact SL(2) sector $(\mathcal{D}, Z)$:

Non-compact spin chains   Tensor structures   Related to 4pt functions

Supersymmetric SU(1|1) sector (Ψ, Z) :

Super spin chains   Tensor structures   Simple result

[Vieira and Wang, 2013]

[Caetano and Fleury, 2014]

One-Loop

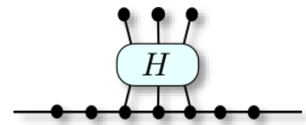
[Gromov, Vieira, 2012]

[Serban, 2012]

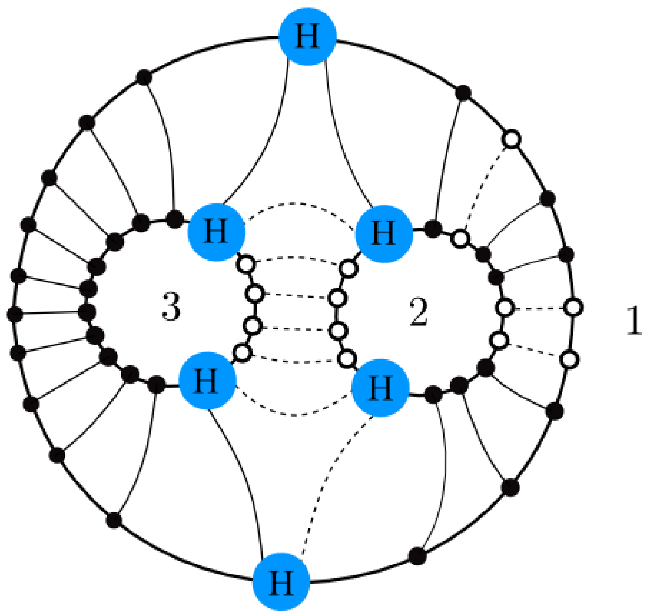
[Y. J. , Leobbert, Kostov, Serban, 2013]

New features

Operator insertions



Long-range spin chain



$$H = g^2 H_2 + g^4 H_4 + \cdots$$

$$H_2 = 2\sum_{k=1}^{L} (1 - \mathrm{P}_{k,k+1})$$

$$H_4 = 2\sum_{k=1}^{L} (2\mathrm{P}_{k,k+1} - \mathrm{P}_{k,k+2} - 3)$$

1. Computed by Feynman diagrams
2. To be taken into account in spin chain

1. Range increases as perturbative order
2. Construct the eigenvectors

The unitary transformation

BDS = **Inhomogeneous XXX** $\left(\theta_k^{\mathrm{BDS}} = 2g \sin \frac{2\pi k}{L} \right)$

$$\mathcal{T}_{\mathrm{BDS}}(u) = S\mathcal{T}_{\mathrm{XXX}}(u;\boldsymbol{\theta}^{\mathrm{BDS}})S^{-1}$$

Construct the eigenstate

$$|\mathbf{u}\rangle_{\mathrm{BDS}} = S|\mathbf{u};\boldsymbol{\theta}^{\mathrm{BDS}}\rangle$$ **standard**

Unitary transformation

$$S = S_{\mathcal{B}} \times S_{\theta}^{-1}$$

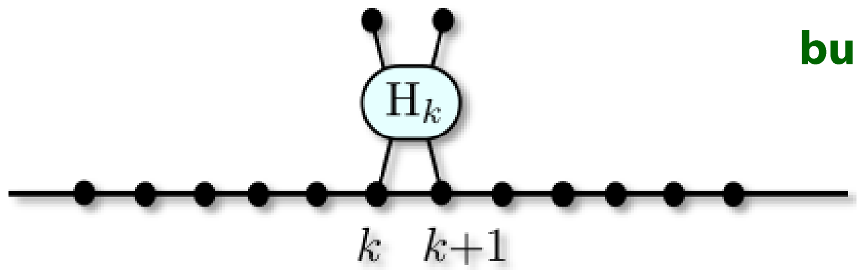
boost **inhomogeneous**

[Bagheer, Beisert, Loebbert 2010]

[Y. J. , Loebbert, Kostov, Serban, 2013]

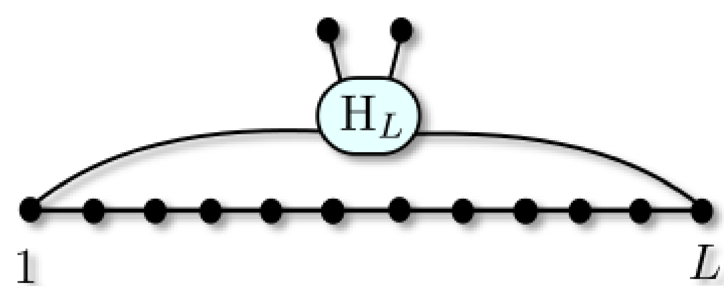
Permutations & Derivatives

$$(\mathrm{I}-\mathrm{P}_{k,k+1})\,|\mathbf{u}\rangle = i\left(\frac{\partial}{\partial\theta_k}-\frac{\partial}{\partial\theta_{k+1}}\right)|\mathbf{u};\boldsymbol{\theta}\rangle|_{\boldsymbol{\theta}=0}$$



bulk

$$(\mathrm{I}-\mathrm{P}_{L,1})\,|\mathbf{u}\rangle = i\left(\frac{\partial}{\partial\theta_L}-\frac{\partial}{\partial\theta_1}\right)|\mathbf{u};\boldsymbol{\theta}\rangle|_{\boldsymbol{\theta}=0} + E(\mathbf{u})|\mathbf{u}\rangle$$



boundary

Result

Fixing one-loop result

$$C_{123}(g^2) = C_{123}^{\mathrm{BDS}}(g^2) + g^2 \delta_{123}$$

$$C_{123}^{\mathrm{BDS}} = \left. \frac{\langle \mathbf{v} \cup \mathbf{z} | \mathbf{u} \rangle_{\boldsymbol{\theta}^{(1)}} \langle \mathbf{w} | \mathbf{z} \rangle_{\boldsymbol{\theta}^{(3)}}}{\sqrt{\langle \mathbf{u} | \mathbf{u} \rangle \langle \mathbf{v} | \mathbf{v} \rangle \langle \mathbf{w} | \mathbf{w} \rangle}} \right|_{\boldsymbol{\theta}^{\mathrm{BDS}}}$$

Suppressed in semi-classical limit

Strong-weak comparison

Take only the BDS part

Compare in the Frolov-Tseytlin limit where $g' = \frac{g}{L} \ll 1$

Weak Coupling

$$F_{123}^{\mathrm{BDS}} \simeq \oint_{\mathcal{C}_{\mathbf{u}} \cup \mathcal{C}_{\mathbf{v}}} \frac{\mathrm{d}u}{2\pi} \mathrm{Li}_2 \left(e^{ip_{\mathbf{u}} + ip_{\mathbf{v}} - iq_{\mathbf{w}}}\right) + \oint_{\mathcal{C}_{\mathbf{w}}} \frac{\mathrm{d}u}{2\pi} \mathrm{Li}_2 \left(e^{ip_{\mathbf{w}} + iq_{\mathbf{u}} - iq_{\mathbf{v}}}\right)$$

Strong Coupling

$$F_{123}^{\mathrm{KK}} \simeq \oint \frac{\mathrm{d}u}{2\pi} \mathrm{Li}_2 \left(e^{ip_1 + ip_2 - ip_3}\right) + \oint \frac{\mathrm{d}u}{2\pi} \mathrm{Li}_2 \left(e^{ip_3 + ip_1 - ip_2}\right) + \oint \frac{\mathrm{d}u}{2\pi} \mathrm{Li}_2 \left(e^{ip_2 + ip_3 - ip_1}\right) + \oint \frac{\mathrm{d}u}{2\pi} \mathrm{Li}_2 \left(e^{ip_1 + ip_2 + ip_3}\right)$$

[Kazama, Komatsu 2013]

Weak Coupling

$$F_{123}^{\mathrm{BDS}} \simeq \oint_{\mathcal{C}_{\mathbf{u}} \cup \mathcal{C}_{\mathbf{v}}} \frac{\mathrm{d}u}{2\pi} \mathrm{Li}_2 \left(e^{ip_{\mathbf{u}} + ip_{\mathbf{v}} - iq_{\mathbf{w}}} \right) + \oint_{\mathcal{C}_{\mathbf{w}}} \frac{\mathrm{d}u}{2\pi} \mathrm{Li}_2 \left(e^{ip_{\mathbf{w}} + iq_{\mathbf{u}} - iq_{\mathbf{v}}} \right)$$

Strong Coupling

$$F_{123}^{\mathrm{KK}} \simeq \oint \frac{\mathrm{d}u}{2\pi} \mathrm{Li}_2 \left(e^{ip_1 + ip_2 - ip_3} \right) + \oint \frac{\mathrm{d}u}{2\pi} \mathrm{Li}_2 \left(e^{ip_3 + ip_1 - ip_2} \right)$$

Perfect match at one-loop !

[Kazama, Komatsu 2015]

Spin Vertex

[Kazama, Komatsu, Nishimura 2014, 2015]
[Y. J. , Petrovskii, Kostov, Serban, 2014]
[Y. J. , Petrovskii, 2014]

String vertex and Spin vertex

3pt functions of BMN string

$$H_{123} = \langle \mathrm{H}_3|1\rangle|2\rangle|3\rangle \sim C_{123}$$

Cubic string vertex

$$\langle \mathrm{H}_3| = \langle 0| \exp\left(-\frac{1}{2} \sum_{i=1}^{8} \sum_{r,s=1}^{3} \sum_{m,n=0}^{\infty} a_m^{(r)i} \tilde{N}_{mn}^{rs} a_n^{(s)i} \right) \mathcal{P}$$

[Spradlin and Volovich 2002]
[Dobashi, Shimada, Yoneya 2003]
[Dobashi and Yoneya 2004]

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Question : Similar object at weak coupling ?

$$C_{123} = \langle \mathrm{V}_3 | 1 \rangle | 2 \rangle | 3 \rangle$$

[Spradlin and Volovich 2002]
[Dobashi, Shimada, Yoneya 2003]
[Dobashi and Yoneya 2004]

String vertex and Spin vertex

3pt functions of BMN string

$$H_{123} = \langle \mathrm{H}_3|1\rangle|2\rangle|3\rangle \sim C_{123}$$

Cubic string vertex

$$\langle \mathrm{H}_3| = \langle 0| \exp\left(-\frac{1}{2}\sum_{i=1}^{8}\sum_{r,s=1}^{3}\sum_{m,n=0}^{\infty} a_m^{(r)i}\,\tilde{N}_{mn}^{rs} a_n^{(s)i}\right)\mathcal{P}$$

Question : Similar object at weak coupling ?

$$C_{123} = \langle \mathrm{V}_3|1\rangle|2\rangle|3\rangle$$

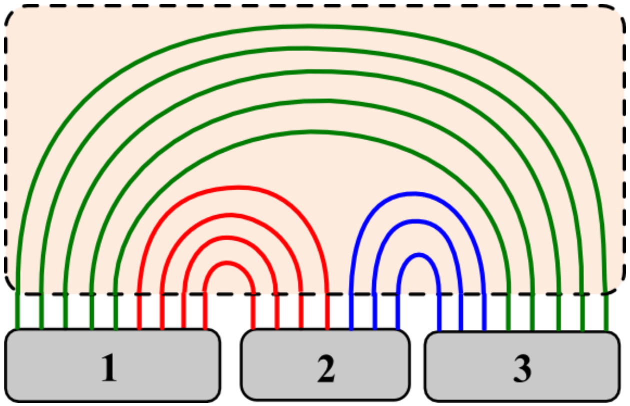
Entanglement entropy ?

$\downarrow$

The spin vertex !

[Spradlin and Volovich 2002]
[Dobashi, Shimada, Yoneya 2003]
[Dobashi and Yoneya 2004]

Construction (leading order)



Factorization :

$$|V_{123}\rangle = |V_{12}\rangle \otimes |V_{23}\rangle \otimes |V_{31}\rangle$$

(*Planar Wick contractions*)

Two-point vertex

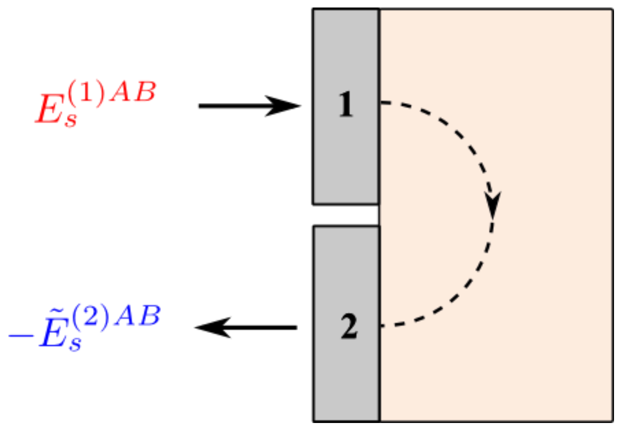
$$|V_{12}\rangle = \exp\left[\sum_{s=1}^{L}\sum_{i=1,2}\left(b_{i,s}^{(1)\dagger}a_{i,s}^{(2)\dagger} - a_{i,s}^{(1)\dagger}b_{i,s}^{(2)\dagger} - d_{i,s}^{(1)\dagger}c_{i,s}^{(2)\dagger} - c_{i,s}^{(1)\dagger}d_{i,s}^{(2)\dagger}\right)\right]|0\rangle \otimes |0\rangle$$

Harmonic oscillators

$$[a_i, a_j^\dagger] = \delta_{ij} \quad [b_i, b_j^\dagger] = \delta_{ij} \quad \{c_i, c_j^\dagger\} = \delta_{ij} \quad \{d_i, d_j^\dagger\} = \delta_{ij}$$

Properties

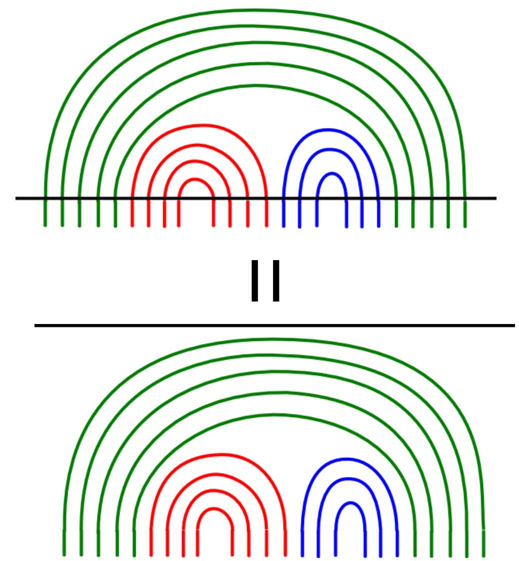
Singlet property



$$\left(E_s^{(1)AB} + \tilde{E}_s^{(2)AB} \right) |V_{12}\rangle = 0$$

1. Symmetry of the spin vertex
2. Translate operators between spin chains

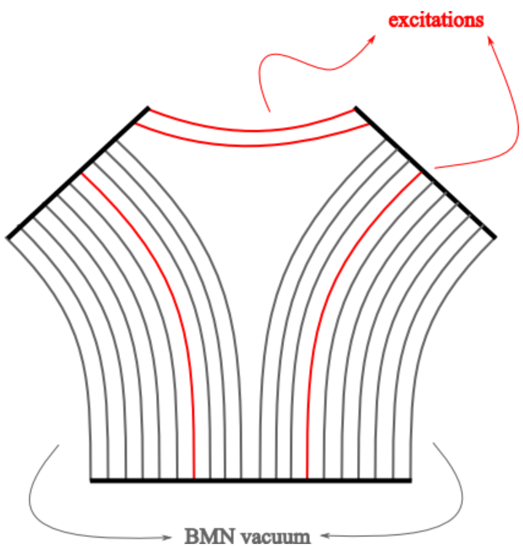
Monodromy condition



$$M_{123}(u)|V_{123}\rangle = |V_{123}\rangle$$

1. Hint of integrability : Yangian symmetry ?
2. Crucial for semi-classical limit

BMN limit



BMN limit

1. Long spin chain with few excitations

2. momentum of excitations are small

$$\left.\begin{aligned} |1\rangle &= \Psi_1\left(\boldsymbol{\partial}_{\boldsymbol{x}}^i\right) \\ |2\rangle &= \Psi_2\left(\boldsymbol{\partial}_{\boldsymbol{y}}^i\right) \\ |3\rangle &= \Psi_3\left(\boldsymbol{\partial}_{\boldsymbol{z}}^i\right) \end{aligned}\right\} \text{External states}$$

Polynomial representation

$$\boxed{\langle V_{123}| = \prod_{k=1}^{L_{12}}(1 + x^i_{L_1-k+1}y^i_k)\prod_{k=1}^{L_{13}}(1 + z^i_{L_3-k+1}x^i_k)\prod_{k=1}^{L_{23}} y^i_{L_2-k+1}z^i_k}$$

Conclusion: Using the replacement $x^i \to a_i,\ \partial/\partial x^i \to a_i^\dagger$ we find the leading order of the bosonic part of the string vertex in light-cone string field theory.

Form Factors

[Bajnok, Janik, Wereszczynski 2014]

[Hollo, Y.J, Petrovskii, 2015]

[Y.J, Petrovskii, 2015]

Towards all loop

General idea : OPE coefficient to Form Factors, and bootstrap.

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Concrete proposals :

- Worldsheet form factor (Klose & Mclaughlin)
- Generalized Neumann coefficients (Bajnok & Janik)
- Hexagon form factors (Basso, Komatsu and Vieira)

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General idea : OPE coefficient to Form Factors, and bootstrap.

Concrete proposals :

- Worldsheet form factor (Klose & McLoughlin)
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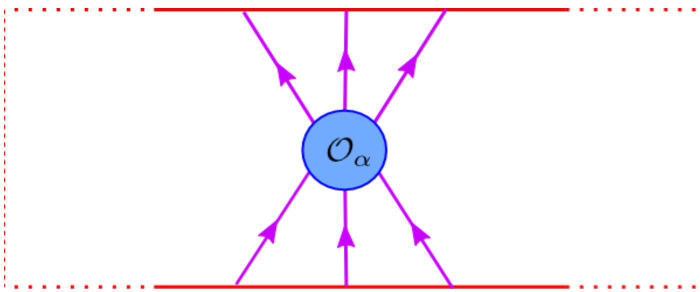
- Hexagon form factors (Basso, Komatsu and Vieira)

State of art :

- Axioms are proposed, but hard to solve
- BKV hexagon is most concrete and powerful

Volume corrections

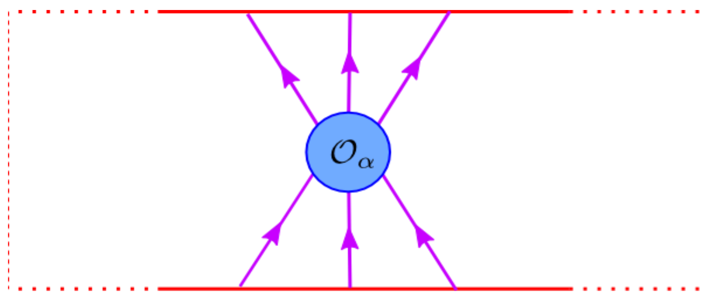
Infinite volume FF



Solution of axioms

Volume corrections

Infinite volume FF



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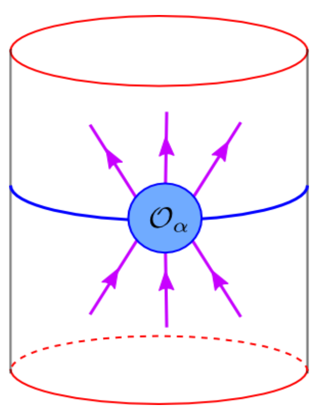
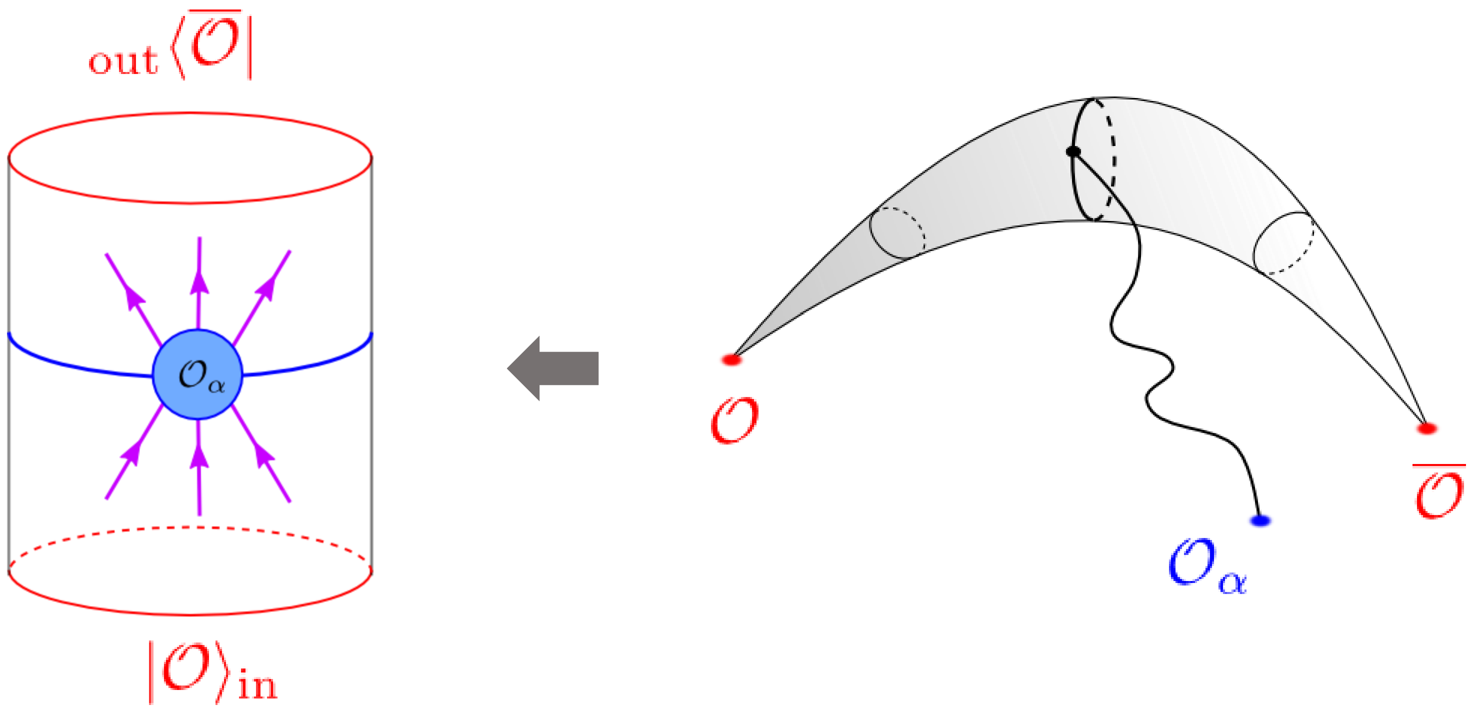


Asymptotic correction $\sim \mathrm{Poly}(\frac{1}{L})$

Wrapping correction $\sim e^{-mL}$

HHL Three-Point Function

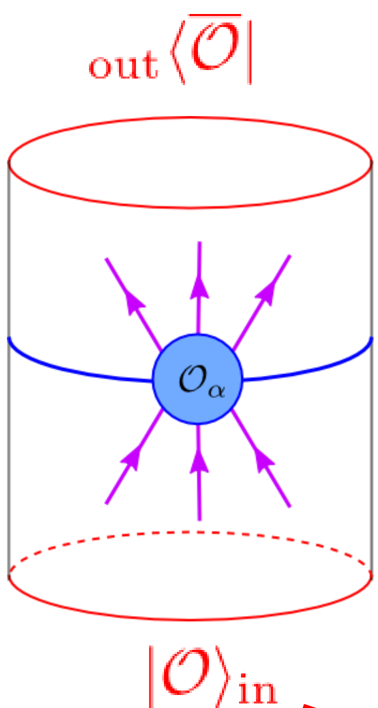
Diagonal Form Factors **HHL Three-Point Functions**

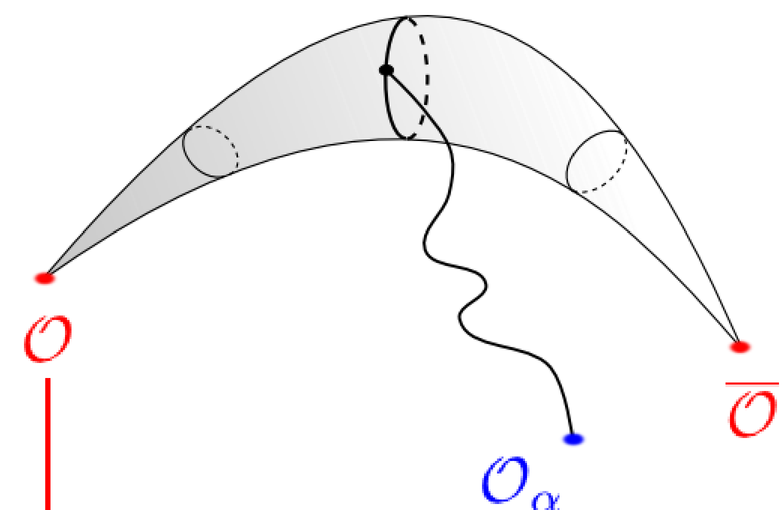


HHL Three-Point Function

Diagonal Form Factors

HHL Three-Point Functions





SUGRA modes

Classical solutions

No back-reaction

Asymptotic correction

BJW Conjecture (asymptotic corrections):

Volume dependence

$$C_{\mathrm{HHL}} = \frac{1}{\rho_N(\mathbf{u})} \sum_{\alpha \cup \bar{\alpha} = \mathbf{u}} f^{\mathcal{O}}(\alpha)\, \rho_N(\bar{\alpha})$$

Infinite volume FF

[Pozsgay and Takacs, 2007]

[Bajnok, Janik, Wereszczynski, 2014]

Asymptotic correction

BJW Conjecture (asymptotic corrections):

Volume dependence

$$C_{\mathrm{HHL}} = \frac{1}{\rho_N(\mathbf{u})} \sum_{\alpha \cup \bar{\alpha} = \mathbf{u}} f^{\mathcal{O}}(\alpha)\, \rho_N(\bar{\alpha})$$

Infinite volume FF

Gaudin determinant (norm of Bethe states)

$$\boxed{\rho_N(\mathbf{u}) = \det_{j,k}\left(\frac{\partial}{\partial u_j}\left(Lp(u_k) - i\sum_{l\neq k}\log S(u_l,u_k)\right)\right)}$$

[Pozsgay and Takacs, 2007]

[Bajnok, Janik, Wereszczynski, 2014]

Check / Proof of the conjecture

Strong coupling

- Several examples (Bajnok, Janik, Wereszcznski)

Weak coupling

- Proof at tree level and one-loop (Hollo, YJ, Petrovskii)
- Done in SU(2) sector, using ABA and QISM

Finite coupling

- Proof up to bridge wrapping (YJ, Petrovskii)
- Use the BKV hexagon formulation
- All rank 1 sectors

Outlook

- Understand wrapping corrections
- Higher loop direct calculation
- Higher rank sectors
- Semi-classical limit including wrapping
- Relation to spectral problem (SoV approach ?)
- 3pt in related theories, ABJM, AdS_3/CFT_2, deformed theories ?