3d Higher Spins Coupled to Scalars

based on hep-th/1505.05887 with G. Lucena-Gomez, E. Skvortsov, M. Taronna
hep-th/1508.04139 with N. Boulanger, E. Skvortsov, M. Taronna
Part 1: Vasiliev Theory: What is it good for?

Part 2: Vasiliev Theory: How does it work?
In this talk we will focus on the bulk physics:

Vasiliev Theory

on $AdS_3$

2d CFT: $\mathcal{W}_N$ - minimal model

Gauge fields:

$\phi_{mn}, \phi_{mnr} \cdots$

One complex scalar:

$\phi$

One parameter family:

$\lambda = \frac{1}{2}$

[Gaberdiel, Gopakumar]
basic open questions in the bulk:

- action?
- degree of locality?
- spectrum?
- quantization?
- how to extract interactions?
Part 1:

Vasiliev Theory: What is it good for?
Free Theory on AdS-background:

$$\square \phi_{m(s)} - \nabla_m \nabla^n \phi_{nm(s-1)} + \frac{1}{2} \nabla_m \nabla_m \phi_{nm(s-2)} - \Lambda m_s^2 \phi_{m(s)} + 2 \Lambda g_{mm} \phi_{m(s-2)n} = 0$$

[Fronsdal ’78]

gauge symmetry: \( \delta \phi_{m(s)} = \nabla_m \xi_{m(s-1)} \)
Example: \( s = 2 \quad \Lambda = 0 \)

\[ \square h_{mm} - \partial_m \partial^n h_{nm} + \frac{1}{2} \partial_m \partial_m h^n_n = 0 \]

But this is \( R^{Lin}_{mm} = 0 \)

\[ g_{mm} = \eta_{mm} + \kappa h_{mm} \]

\[ R_{mm} = \kappa R^{Lin}_{mm} + \mathcal{O}(\kappa^2) \]
Can one construct fully non-linear equations of motion for HS fields?
Vasiliev can !!!

higher spin particles?

Him
\[ \Box \phi_{m(s)} - \nabla_m \nabla^n \phi_{nm(s-1)} + \cdots = j_{m(s)}(\{\phi\}) \]

Vasiliev Equations

We cranked the handle up to second order in perturbations around AdS.

\[ \phi_{mn} = g^{AdS}_{mn} + \kappa h^{(1)}_{mn} + \kappa^2 h^{(2)}_{mn} + \mathcal{O}(\kappa^3) \]

\[ \phi_{m(s)} = \kappa \phi^{(1)}_{m(s)} + \kappa^2 \phi^{(2)}_{m(s)} + \mathcal{O}(\kappa^3) \text{ for } s \neq 2 \]
The result for spin 2:

\[ \square \phi_m(s) + \ldots \bigg|_{\phi\phi} = \sum_{k=0}^{s} \sum_{l=0}^{\infty} \left( a_{l,k} \nabla m(s-k)n(l) \phi \nabla m(k)^{n(l)} \phi + \text{traces} \right) \]

\[ a_{l,k} \neq 0 \]

for generic value of \( l \)

“pseudo-local”

Restrict to scalar sector
(independently conserved)
Metsaev [2006]: Up to field redefinitions the spin-s current involving two scalars contains only s derivatives.

\[ \Box \phi_{m(s)} + \cdots \bigg|_{\phi\phi} = \sum_{k=0}^{s} \left( b_k \nabla_{m(s-k)} \phi \nabla_{m(k)} \phi + \text{traces} \right) \]

Can also be determined from symmetry arguments


We fixed the complete cubic action!
Construct a field redefinition to relate the two results:

\[ \square \phi_{m(s)} + \ldots \bigg|_{\phi\phi} = \sum_{k=0}^{s} \sum_{l=0}^{\infty} (a_{l,k} \nabla_{m(s-k)n(l)}\phi \nabla_{m(k)^{n(l)}}\phi + \text{traces}) \]

\[ \nabla_{m(s-k)n(L)}\phi \nabla_{m(k)^{n(L)}}\phi \]

redefinition of \( \phi_{m(s)} \) with \#\( \nabla < s + L \)

\[ \sum_{l=0}^{L-1} \#_{l} \nabla_{m(s-k)n(l)}\phi \nabla_{m(k)^{n(l)}}\phi \]

\[ \ldots \]

\[ C_{L} \nabla_{m(s-k)}\phi \nabla_{m(k)}\phi \]
So in total we get:

\[ \Box \phi_{m(s)} + \cdots = \sum_{k=0}^{s} \sum_{l=0}^{\infty} \left( a_{l,k} \nabla_{m(s-k)n(l)} \phi \nabla_{m(k)} n(l) \phi + \text{traces} \right) \]

\[ \Box \phi_{m(s)} + \cdots = \sum_{k=0}^{s} \sum_{l=0}^{\infty} C_{l} a_{l,k} \nabla_{m(s-k)} \phi \nabla_{m(k)} \phi + \text{traces} \]

divergent!

[N. Boulanger, P.K, E.Skvorstov, M. Taronna]
[E.Skvorstov, M. Taronna]
Theorem: Any source term $j_m(s)$ can be removed by a pseudo-local field redefinition.

[Prokushkin, Vasiliev '00]
**Idea:** Use AdS/CFT as a consistency check

\[ S = \int \frac{1}{2} \Phi (\Box - m^2) \Phi + \Psi (\Box - M^2) \Psi - a_0 \Phi^2 \Psi - a_1 (\partial \Phi)^2 \Psi + \ldots \]

\[ \Psi \rightarrow \Psi + \frac{1}{2} a_1 \Phi^2 \]

\[ S' = \int \frac{1}{2} \Phi (\Box - m^2) \Phi + \Psi (\Box - M^2) \Psi - \left( a_0 + \frac{1}{2} a_1 (2m^2 - M^2) \right) \Phi^2 \Psi + \ldots \]

\[ = \langle O_\Phi O_\Phi O_\psi \rangle_{CFT} \text{ is left invariant.} \]

[Freedman, Mathur, Matusis, Rastelli '98]
3pt calculation using source term before and after field redefinition leads to the same result.

\[ \square \phi_m(s) + \cdots = \sum_{k=0}^{s} \sum_{l=0}^{\infty} \left( a_{l,k} \nabla_m(s-k) n(l) \phi \nabla_m(k) n(l) \phi + \text{traces} \right) \]

[Skvortsov, Taronna]
But this is puzzling:

\[ \square \phi_m(s) + \cdots = j_m(s)(\phi, \phi) \]

\[ \square \phi + \cdots = j'(\phi_m(s), \phi) \]

[Ammon, Kraus, Perlmutter]
[Giombi, Yin]
Summary of Part 1:

- Extraction of $j_m(s)$ to second order in perturbations around AdS.
- Criterion for allowed field redefinitions is found.
- 3pt function calculated from gauge fields diverges.
Part 2:

Vasiliev Theory: How does it work?

Step 1: Linearised Equations
\[ y_\alpha \quad \alpha \in \{0, 1\} \]

\[ y_\alpha y_\beta = y_\beta y_\alpha \]

obeying:

\[ \varphi^2 = 1 \]

\[ \varphi y_\alpha = y_\alpha \varphi \]

Star product:

\[ (f \star g)(y) = f(y) e^{-i \bar{\partial}_\alpha \epsilon^{\alpha \beta} \partial_\beta} g(y) \]

Rep of AdS isometry algebra:

\[ L_{\alpha \beta} \sim y_{(\alpha} \star y_{\beta)} \quad P_{\alpha \beta} = \varphi L_{\alpha \beta} \]

\[ [L_{\alpha \beta}, L_{\alpha' \beta'}]_\star = \epsilon_{\alpha \alpha'} L_{\beta \beta'} + \ldots \]

\[ [L_{\alpha \beta}, P_{\alpha' \beta'}]_\star = \epsilon_{\alpha \alpha'} P_{\beta \beta'} + \ldots \]

\[ [P_{\alpha \beta}, P_{\alpha' \beta'}]_\star = \epsilon_{\alpha \alpha'} L_{\beta \beta'} + \ldots \]
AdS - background:

\[ \bar{\Omega} = \bar{\omega}^{\alpha\beta} L_{\alpha\beta} + \bar{e}^{\alpha\beta} P_{\alpha\beta} \sim (\bar{\omega}^{\alpha\beta} + \varphi \bar{e}^{\alpha\beta}) y_{(\alpha \ast y_{\beta})} \]

Obeying the equation of motion:

\[ d\bar{\Omega} - \bar{\Omega} \wedge \ast \bar{\Omega} = 0 \]

Metric is obtained from:

\[ g^{\text{AdS}}_{mn} = \bar{e}^{\alpha\beta}_{m} \bar{e}_{\alpha\beta n} \]
A natural generalisation to HS case:

\[ \Omega = \sum_s \left( \omega^{\alpha(2s)} + \varphi e^{\alpha(2s)} \right) y_{(\alpha_1 \star \cdots \star y_{\alpha_{2s}})} \]

Obeying the equation of motion:

\[ D_\Omega \Omega = d\Omega - \bar{\Omega} \wedge *\Omega - \Omega \wedge *\bar{\Omega} = 0 \]

\[ D_\Omega F := dF - \bar{\Omega} \wedge *F + (-1)^{|F|} F \wedge *\bar{\Omega} \]

\[ = \nabla F - \bar{e} \wedge *F + (-1)^{|F|} F \wedge *\bar{e} \]

Gauge symmetry:

\[ \delta \Omega = D_\Omega \xi(y, \varphi|x) \]
Spin $s$ field is obtained by

$$\phi_m(s) = e^{\alpha(2s)}\bar{e}_m\alpha\alpha\cdots\bar{e}_m\alpha\alpha$$

Solve torsion constraint

$$\omega = \omega(e)$$

Fronsdal equation:

$$\square\phi_m(s) + \cdots = 0$$
Scalar field

$$\Box_{AdS} \phi = m^2 \phi$$

Has to be rewritten in “unfolded” form:

$$\nabla C - \bar{e} \wedge \ast C - C \wedge \ast \bar{e} = 0$$

$$C(y) = \sum_s C_{\alpha(s)} \, y^{\alpha_1} \ast \cdots \ast y^{\alpha_s}$$

$$C(y = 0) = \phi$$

$$C_{\alpha(s)} \sim (\bar{e}^m_{\alpha\alpha})^s \phi$$
\[ D_\Omega C \neq \nabla C - \bar{e} \wedge \star C - C \wedge \star \bar{e} = 0 \]

\[ B = C \psi \quad \text{with} \quad \psi \varphi = -\varphi \psi \quad \psi^2 = 1 \]

\[ D_\Omega B = D_\Omega (C \psi) = (\nabla C) \psi - \bar{e} \wedge \star C \psi + C \psi \wedge \star \bar{e} \]

\[ = (\nabla C - \bar{e} \wedge \star C - C \wedge \star \bar{e}) \psi = 0 \]

\[ \bar{e} \sim \varphi e^{\alpha \beta} y_\alpha \star y_\beta \]

“Twisted adjoint representation”
Summary of free equations:

\[ D_\Omega \Omega = 0 \quad \delta \Omega = D_\Omega \xi \]
\[ D_\Omega B = 0 \quad \delta B = 0 \]

There is a natural generalisation:

\[ B = C\psi \quad \Rightarrow \quad B = C\psi + C^{tw} \]
\[ \Omega \quad \Rightarrow \quad \Omega = \omega + \omega^{tw} \psi \]

In fact Vasiliev equations require these additional twisted fields
Twisted fields can be consistently be set to zero up to 2nd order perturbations around AdS.

Step 2: Non-linear equations (= Vasiliev equations)
More formalism:

Additional variable $z_\alpha$ commutes with $y_\alpha, \varphi, \psi$

$$(f \star g)(y,z) = f(y,z)e^{-i(\partial_y + \partial_z)_{\alpha}(\partial_y - \partial_z)^{\alpha}}g(y,z)$$

e.g. $z_\alpha \star f(y) = (z_\alpha + i\partial_{\alpha}^y)f(y)$
All fields depend on all variables:

\[ B(y, \psi, \varphi) \quad \rightarrow \quad \hat{B}(z, y, \psi, \varphi) \]

\[ \Omega(y, \psi, \varphi) \quad \rightarrow \quad \hat{\Omega}(z, y, \psi, \varphi) \]
Vasiliev equations:

\[ D_\Omega \hat{\Omega} = \hat{\Omega} \wedge \star \hat{\Omega} \]
\[ D_\Omega \hat{B} = [\hat{W}, \hat{\Omega}]_\star \]

\[ \partial^z_\alpha \hat{\Omega} = \ldots \]
\[ \partial^z_\alpha \hat{B} = \ldots \]
\[ \hat{\Omega} = \Omega(y) + z_\alpha g^\alpha(\Omega, B) \]

First equation is then evaluated at \( z=0 \):

\[ D_\Omega \hat{\Omega} = \hat{\Omega} \land \star \hat{\Omega} \]
\[ D_\Omega \hat{B} = [\hat{W}, \hat{\Omega}]_\star \]

\[ \partial^z \hat{\Omega} = \ldots \]
\[ \partial^z \hat{B} = \ldots \]

\[ z_\alpha \ast f(y) = (z_\alpha + i\partial^\alpha_y) f(y) \]

\( z \) encodes interaction!
Possible subtle points:

- metric-like \rightarrow \text{ frame-like}
  \[ \phi_m(s) = e_m^{(2s)} \bar{e}_m \alpha \alpha \cdots \bar{e}_m \alpha \alpha \]

- Schwinger-Fock gauge:
  \[ \xi(z, y) \rightarrow \xi(y) \]
Conclusions

• We extracted interactions from Vasiliev equations up to 2nd order around AdS.

• We could clarify:
  • Twisted fields decouple to this order
  • cubic action by symmetry
  • class of allowed field redefinitions

• New puzzle: Divergences in 3pt function
Questions?