3d Higher Spins Coupled to Scalars

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based on hep-th/1505.05887 with G.Lucena-Gomez, E.Skvortsov, M.Taronna
hep-th/1508.04139 with N.Boulanger, E.Skvortsov, M.Taronna

Pan's Plan:

Part 1: Vasiliev Theory: What is it good for?

Part 2: Vasiliev Theory: How does it work?

In this talk we will focus on the bulk physics:

gauge fields:

$\phi_{mn}, \phi_{mnr} \cdots$

one complex scalar:

ϕ

one parameter family:

$\lambda = \frac{1}{2}$

Vasiliev Theory on AdS_3

2d CFT:

$\mathcal{W}_N$ - minimal model

[Gaberdiel, Gopakumar]

basic open questions in the bulk:

action?

degree of locality?

spectrum?

quantization?

how to extract interactions?

Part 1:

Vasiliev Theory: What is it good for?

Free Theory on AdS-background:

$$\Box\phi_{m(s)} - \nabla_m \nabla^n \phi_{nm(s-1)} + \frac{1}{2}\nabla_m \nabla_m \phi^n{}_{nm(s-2)} - \Lambda m_s^2 \phi_{m(s)} + 2\Lambda g_{mm}\phi_{m(s-2)n}{}^n = 0$$

[Fronsdal '78]

gauge symmetry: $\delta\phi_{m(s)} = \nabla_m \xi_{m(s-1)}$

Example: $s = 2$ $\Lambda = 0$

$$\Box h_{mm} - \partial_m \partial^n h_{nm} + \frac{1}{2} \partial_m \partial_m h^n{}_n = 0$$

But this is $R^{Lin}_{mm} = 0$

$$g_{mm} = \eta_{mm} + \kappa h_{mm}$$

$$\downarrow$$

$$R_{mm} = \kappa R^{Lin}_{mm} + \mathcal{O}(\kappa^2)$$

Can one construct fully non-linear equations of motion for HS fields?

Vasiliev can !!!

???

Him

higher spin particles?

$$\Box\phi_{m(s)} - \nabla_m \nabla^n \phi_{nm(s-1)} + \cdots = j_{m(s)}(\{\phi\})$$

We cranked the handle up to second order in perturbations around AdS.

$$\phi_{mn} = g^{AdS}_{mn} + \kappa\, h^{(1)}_{mn} + \kappa^2\, h^{(2)}_{mn} + \mathcal{O}(\kappa^3)$$

$$\phi_{m(s)} = \kappa\, \phi^{(1)}_{m(s)} + \kappa^2\, \phi^{(2)}_{m(s)} + \mathcal{O}(\kappa^3) \quad \text{for} \quad s \neq 2$$

The result for spin 2:

$$\Box\phi_{m(s)} + \ldots \big|_{\phi\phi} = \sum_{k=0}^{s}\sum_{l=0}^{\infty}\left(a_{l,k}\nabla_{m(s-k)n(l)}\phi\nabla_{m(k)}{}^{n(l)}\phi + \mathbf{traces}\right)$$

$$a_{l,k} \neq 0$$

for generic value of l

"pseudo-local"

Restrict to scalar sector
(independently conserved)

Metsaev [2006]: Up to field redefinitions the spin-s current involving two scalars contains only s derivatives.

$$\Box\phi_{m(s)} + \ldots\big|_{\phi\phi} = \sum_{k=0}^{s}\left(b_k \nabla_{m(s-k)}\phi\nabla_{m(k)}\phi + \textbf{traces}\right)$$

Can also be determined from symmetry arguments

[P.K, G. Lucena Gomez, E.Skvortsov, M. Taronna]

We fixed the complete cubic action!

Construct a field redefinition to relate the two results:

$$\Box\phi_{m(s)} + \ldots\Big|_{\phi\phi} = \sum_{k=0}^{s}\sum_{l=0}^{\infty}\left(a_{l,k}\nabla_{m(s-k)n(l)}\phi\nabla_{m(k)}{}^{n(l)}\phi + \textbf{traces}\right)$$

$$\nabla_{m(s-k)n(L)}\phi\nabla_{m(k)}{}^{n(L)}\phi$$

redefinition of $\phi_{m(s)}$ with $\#\nabla < s+L$

$$\longrightarrow \sum_{l=0}^{L-1}\#_l\nabla_{m(s-k)n(l)}\phi\nabla_{m(k)}{}^{n(l)}\phi$$

$$\longrightarrow \quad \ldots.$$

$$\longrightarrow \quad C_L\nabla_{m(s-k)}\phi\nabla_{m(k)}\phi$$

So in total we get:

$$\Box\phi_{m(s)} + \cdots = \sum_{k=0}^{s}\sum_{l=0}^{\infty}\left(a_{l,k}\nabla_{m(s-k)n(l)}\phi\nabla_{m(k)}{}^{n(l)}\phi + \textbf{traces}\right)$$

↓ ?

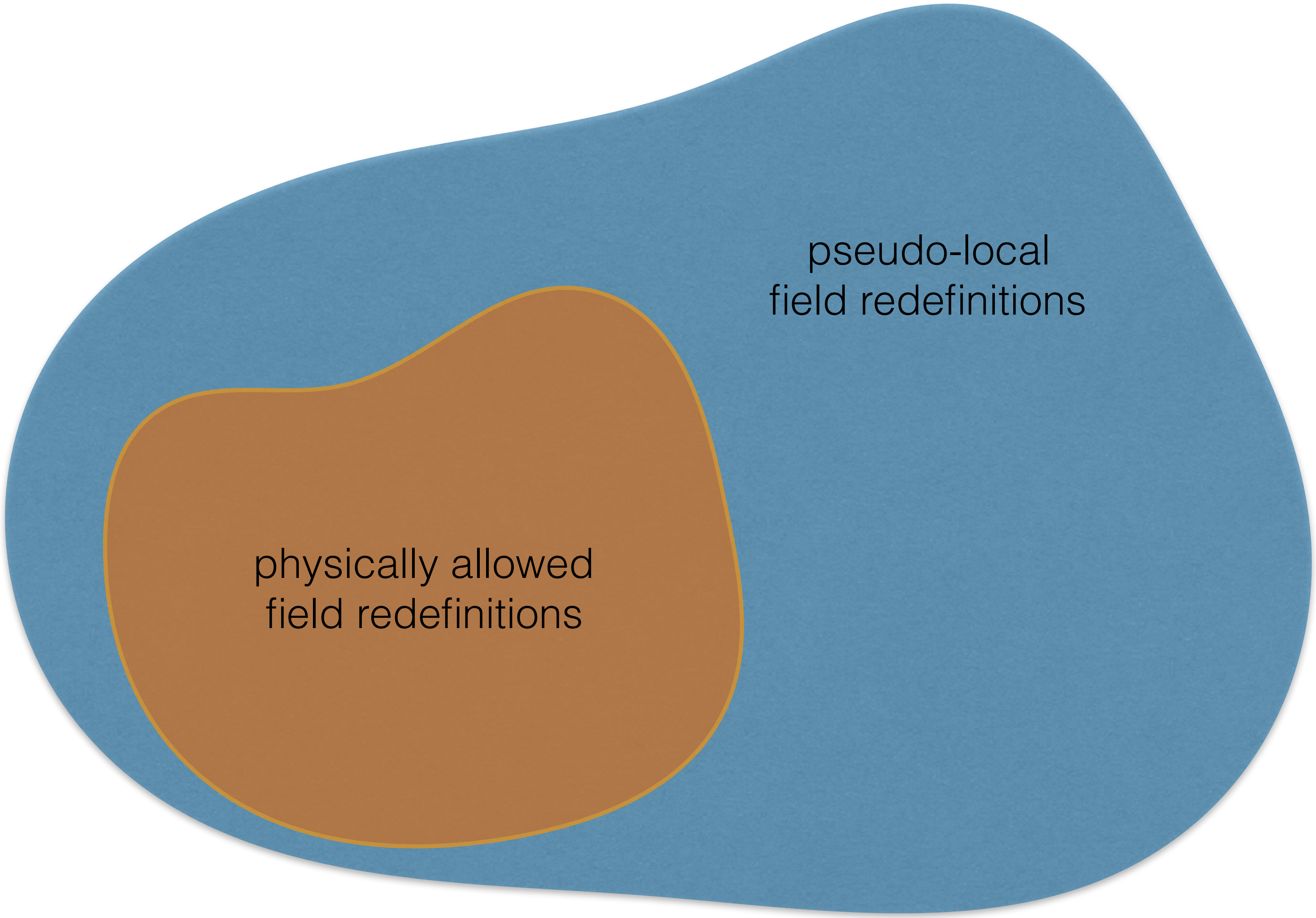
$$\Box\phi_{m(s)} + \cdots = \sum_{k=0}^{s}\sum_{l=0}^{\infty} C_l\, a_{l,k}\nabla_{m(s-k)}\phi\nabla_{m(k)}\phi + \textbf{traces}$$

divergent!

[N. Boulanger, P.K, E.Skvortsov, M. Taronna]

[E.Skvortsov, M. Taronna]

$$\Box\phi_{m(s)} + \cdots = j_{m(s)}$$



Theorem: Any source term $j_{m(s)}$ can be removed by a pseudo-local field redefinition.

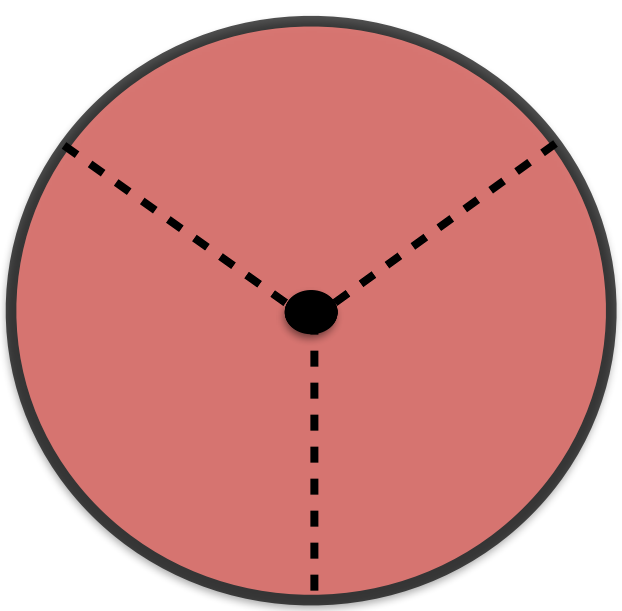
[Prokushkin, Vasiliev '00]
[P.K, G. Lucena Gomez, E.Skvortsov, M. Taronna]

Idea: Use AdS/CFT as a consistency check

$$S = \int \frac{1}{2}\Phi(\Box - m^2)\Phi + \Psi(\Box - M^2)\Psi - a_0\Phi^2\Psi - a_1(\partial\Phi)^2\Psi + \ldots$$

$$\Psi \to \Psi + \frac{1}{2}a_1\Phi^2$$

$$S' = \int \frac{1}{2}\Phi(\Box - m^2)\Phi + \Psi(\Box - M^2)\Psi - \left(a_0 + \frac{1}{2}a_1(2m^2 - M^2)\right)\Phi^2\Psi + \ldots$$



$= \langle O_\Phi O_\Phi O_\psi \rangle_{CFT}$ is left invariant.

[Freedman, Mathur, Matusis, Rastelli '98]

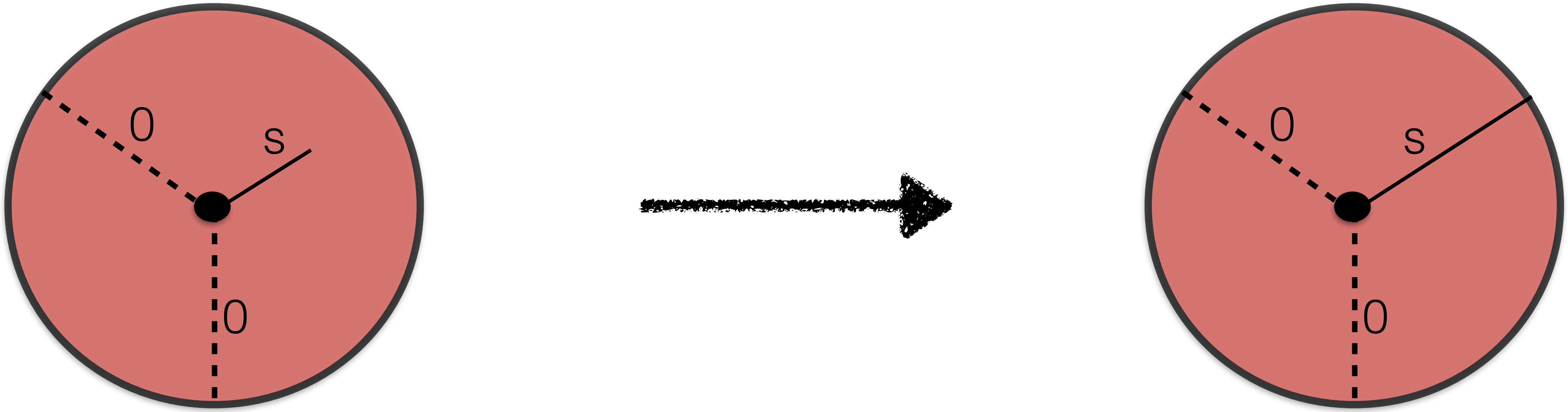
3pt calculation using source term before and after field redefinition leads to the same result.

$$\Box\phi_{m(s)} + \cdots = \sum_{k=0}^{s}\sum_{l=0}^{\infty}\left(a_{l,k}\nabla_{m(s-k)n(l)}\phi\nabla_{m(k)}{}^{n(l)}\phi + \textbf{traces}\right)$$

$$\Box\phi_{m(s)} + \cdots = \sum_{k=0}^{s}\sum_{l=0}^{\infty}\left(C_l\, a_{l,k}\nabla_{m(s-k)}\phi\nabla_{m(k)}\phi + \textbf{traces}\right)$$

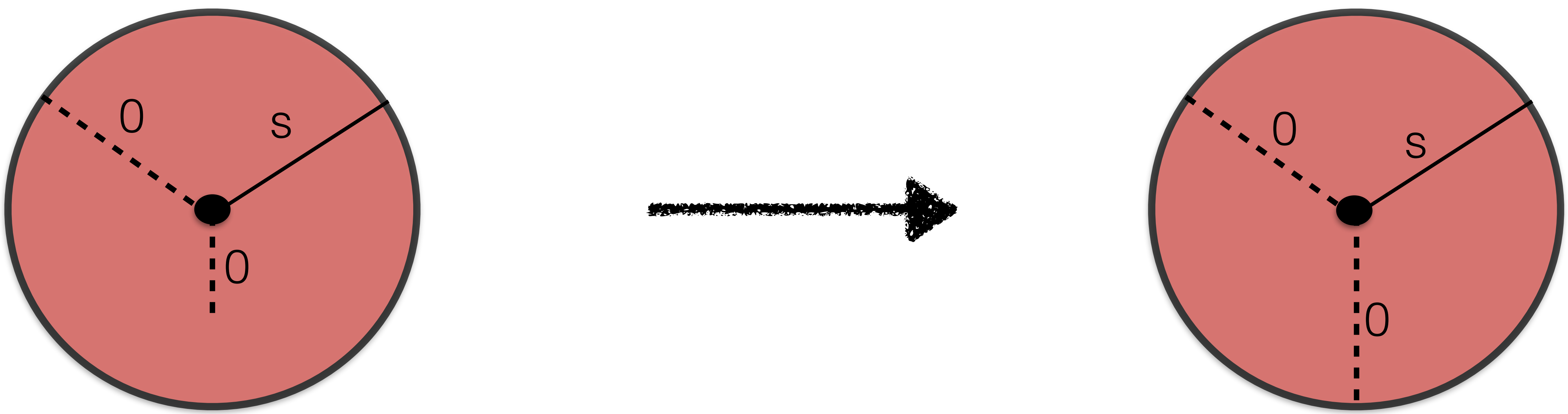
[Skvortsov, Taronna]

But this is puzzling:



$\Box\phi_{m(s)} + \cdots = j_{m(s)}(\phi, \phi)$

DIVERGES!



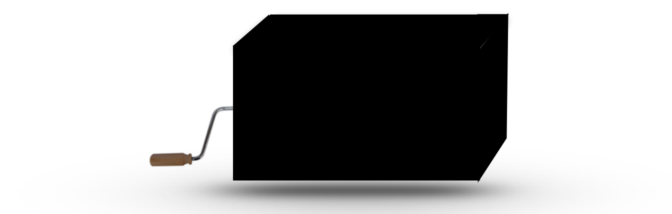
$\Box\phi + \cdots = j'(\phi_{m(s)}, \phi)$

FINITE!

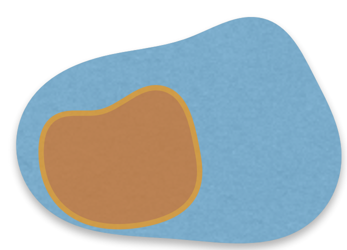
[Ammon, Kraus, Perlmutter]
[Giombi, Yin]

Summary of Part 1:

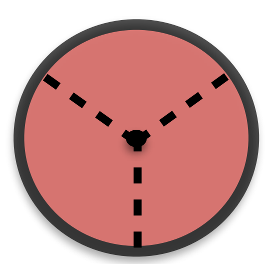
- Extraction of $j_{m(s)}$ to second order in perturbations around AdS.



- Criterion for allowed field redefinitions is found.



- 3pt function calculated from gauge fields diverges.



Part 2:

Vasiliev Theory: How does it work?

[Hopefully out soon: “Lectures on Minimal Model Holography”,
A. Campoleoni, S. Fredenhagen, P.K., G. Lucena Gomez]

Step1: Linearised Equations

$y_\alpha \qquad \alpha \in \{0, 1\}$

φ

obeying:

$$y_\alpha y_\beta = y_\beta y_\alpha$$

$$\varphi^2 = 1$$

$$\varphi y_\alpha = y_\alpha \varphi$$

Star product:

$$(f \star g)(y) = f(y)\, e^{-i \overleftarrow{\partial}_\alpha \overbrace{\epsilon^{\alpha\beta} \overrightarrow{\partial}_\beta}^{=: \overrightarrow{\partial}^\alpha}}\, g(y)$$

Rep of AdS isometry algebra: $L_{\alpha\beta} \sim y_{(\alpha} \star y_{\beta)} \qquad P_{\alpha\beta} = \varphi L_{\alpha\beta}$

$$[L_{\alpha\beta}, L_{\alpha'\beta'}]_\star = \epsilon_{\alpha\alpha'} L_{\beta\beta'} + \dots$$

$$[L_{\alpha\beta}, P_{\alpha'\beta'}]_\star = \epsilon_{\alpha\alpha'} P_{\beta\beta'} + \dots$$

$$[P_{\alpha\beta}, P_{\alpha'\beta'}]_\star = \epsilon_{\alpha\alpha'} L_{\beta\beta'} + \dots$$

AdS - background:

$$\bar{\Omega} = \bar{\omega}^{\alpha\beta} L_{\alpha\beta} + \bar{e}^{\alpha\beta} P_{\alpha\beta} \sim \left(\bar{\omega}^{\alpha\beta} + \varphi \bar{e}^{\alpha\beta}\right) y_{(\alpha} \star y_{\beta)}$$

Obeying the equation of motion:

$$d\bar{\Omega} - \bar{\Omega} \wedge \star \bar{\Omega} = 0$$

Metric is obtained from:

$$g^{AdS}_{mn} = \bar{e}^{\alpha\beta}_m \bar{e}_{\alpha\beta\, n}$$

A natural generalisation to HS case:

$$\Omega = \sum_s \left(\omega^{\alpha(2s)} + \varphi e^{\alpha(2s)}\right) y_{(\alpha_1} \star \cdots \star y_{\alpha_{2s})}$$

Obeying the equation of motion:

$$D_\Omega \Omega = d\Omega - \bar{\Omega} \wedge \star\Omega - \Omega \wedge \star\bar{\Omega} = 0$$

$$D_\Omega F := dF - \bar{\Omega} \wedge \star F + (-1)^{|F|} F \wedge \star\bar{\Omega}$$
$$= \nabla F - \bar{e} \wedge \star F + (-1)^{|F|} F \wedge \star\bar{e}$$

Gauge symmetry:

$$\delta\Omega = D_\Omega \xi(y, \varphi|x)$$

Spin s field is obtained by

$$\phi_{m(s)} = e_m^{\alpha(2s)} \bar{e}_{m\,\alpha\alpha} \ldots \bar{e}_{m\,\alpha\alpha}$$

$$D_\Omega \Omega = 0$$

Solve torsion contraint

$$\omega = \omega(e)$$

Fronsdal equation:

$$\Box \phi_{m(s)} + \cdots = 0$$

Scalar field

$$\Box_{AdS}\phi = m^2\phi$$

Has to be rewritten in “unfolded” form:

$$\nabla C - \bar{e} \wedge \star C - C \wedge \star \bar{e} = 0$$

$$C(y) = \sum_s C_{\alpha(s)}\, y^{\alpha_1} \star \cdots \star y^{\alpha_s}$$

$$C(y=0) = \phi$$

$$C_{\alpha(s)} \sim (\bar{e}^m_{\alpha\alpha})^s \phi$$

$$D_\Omega C \neq \; \nabla C - \bar{e} \wedge \star C - C \wedge \star \bar{e} = 0$$

$$B = C\psi \quad \text{with} \quad \psi\varphi = -\varphi\psi \qquad \psi^2 = 1$$

$$D_\Omega B = D_\Omega \left(C\psi\right) = (\nabla C)\psi - \bar{e} \wedge \star C\psi + C\psi \wedge \star \bar{e}$$

$$= (\nabla C - \bar{e} \wedge \star C - C \wedge \star \bar{e})\psi = 0$$

$$\bar{e} \sim \varphi \, e^{\alpha\beta} \, y_\alpha \star y_\beta$$

"Twisted adjoint representation"

Summary of free equations:

$$D_\Omega \Omega = 0 \qquad \delta\Omega = D_\Omega \xi$$

$$D_\Omega B = 0 \qquad \delta B = 0$$

There is a natural generalisation:

$$B = C\psi \longrightarrow B = C\psi + C^{tw}$$

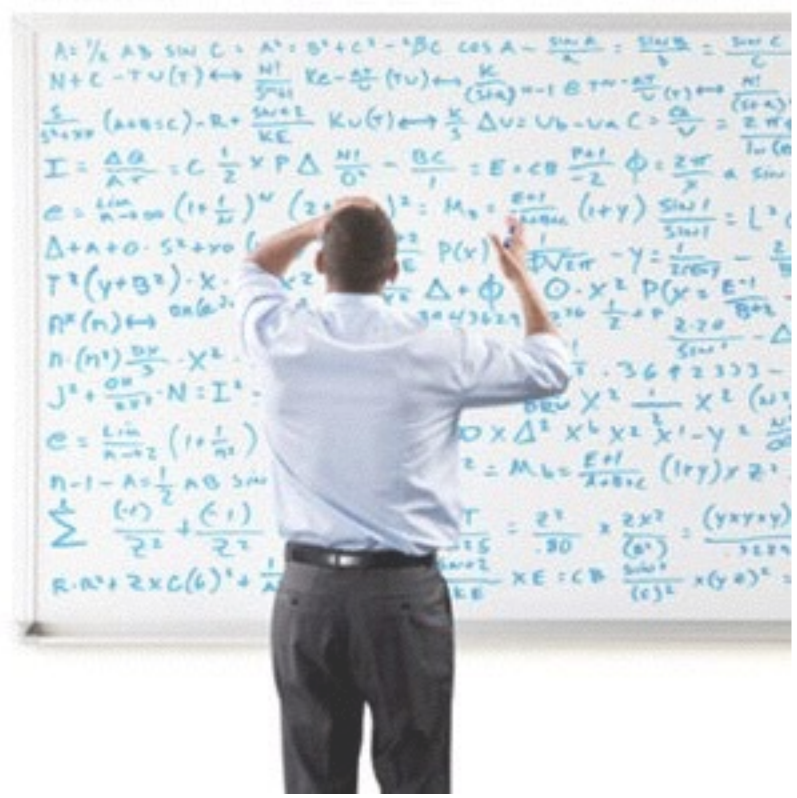
$$\Omega \longrightarrow \Omega = \omega + \omega^{tw}\psi$$

In fact Vasiliev equations require these additional twisted fields

Twisted fields can be consistently be set to zero up to 2nd order perturbations around AdS.

[P.K, G. Lucena Gomez, E.Skvortsov, M. Taronna]

Step 2: Non-linear equations (= Vasiliev equations)



More formalism:

Additional variable z_α commutes with y_α, φ, ψ

$$(f \star g)(y,z) = f(y,z) e^{-i(\overleftarrow{\partial}_y + \overleftarrow{\partial}_z)_\alpha (\overrightarrow{\partial}_y - \overrightarrow{\partial}_z)^\alpha} g(y,z)$$

e.g. $z_\alpha \star f(y) = (z_\alpha + i\partial^y_\alpha) f(y)$

All fields depend on all variables:

$$B(\mathbf{y}, \psi, \varphi) \longrightarrow \hat{B}(\mathbf{z}, \mathbf{y}, \psi, \varphi)$$

$$\Omega(\mathbf{y}, \psi, \varphi) \longrightarrow \hat{\Omega}(\mathbf{z}, \mathbf{y}, \psi, \varphi)$$

Vasiliev equations:

$$D_\Omega \hat{\Omega} = \hat{\Omega} \wedge \star \hat{\Omega}$$

$$D_\Omega \hat{B} = [\hat{W}, \hat{\Omega}]_\star$$

$$\partial^z_\alpha \hat{\Omega} = \ldots$$

$$\partial^z_\alpha \hat{B} = \ldots$$

$$D_\Omega \hat{\Omega} = \hat{\Omega} \wedge \star \hat{\Omega}$$

$$D_\Omega \hat{B} = [\hat{W}, \hat{\Omega}]_\star$$

$$\hat{\Omega} = \Omega(y) + z_\alpha g^\alpha(\Omega, B)$$

$$\partial^z_\alpha \hat{\Omega} = \ldots$$

$$\partial^z_\alpha \hat{B} = \ldots$$

First equation is then evaluated at z=0:

$$\longrightarrow \quad D_\Omega \Omega = F(\Omega, B)$$

$$z_\alpha \star f(y) = (z_\alpha + i\partial^y_\alpha) f(y)$$

z encodes interaction!

Possible subtle points:

- metric-like $\longrightarrow$ frame-like

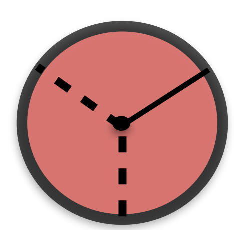
$$\phi_{m(s)} = e_m^{\alpha(2s)} \bar{e}_{m\,\alpha\alpha} \ldots \bar{e}_{m\,\alpha\alpha}$$

- Schwinger-Fock gauge:

$$\xi(z, y) \longrightarrow \xi(y)$$

Conclusions

- We extracted interactions from Vasiliev equations up to 2nd order around AdS.
- We could clarify:
    - Twisted fields decouple to this order
    - cubic action by symmetry
    - class of allowed field redefinitions
- New puzzle: Divergences in 3pt function



Questions?