Amplitudes, Form Factors and the Dilatation Operator of $\mathcal{N} = 4$ SYM theory

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[1410.8485] with D. Nandan, C. Sieg and G. Yang

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2. Gauge-invariant local composite operators
3. Form factors in the free theory
4. One-loop corrections
5. Divergences and the dilatation operator
6. Two-loop Konishi form factor
7. Conclusions and outlook

Motivation to study $\mathcal{N} = 4$ SYM theory

- Gain general understanding of gauge theories
- Develop new techniques for Standard Model calculations
- Strong coupling description via the AdS/CFT correspondence
- Integrability
- "Hydrogen atom of the 21$^{\text{st}}$ century"

Motivation to study form factors (1)

$\mathcal{A}(1,\dots,n)$
$= \langle 1,\dots,n|0\rangle$

$\mathcal{C}_{\mathcal{O}_1\dots\mathcal{O}_n}(x_1,\dots,x_n)$
$= \langle 0|\mathcal{O}_1(x_1)\dots\mathcal{O}_n(x_n)|0\rangle$

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$$\mathcal{F}_{\mathcal{O}}(1,\dots,n;x) = \langle 1,\dots,n|\mathcal{O}(x)|0\rangle$$

Picture of a bridge

(Removed in the online version for copyright reasons)

$\Rightarrow$ Form factors as bridge between purely on-shell amplitudes and purely off-shell correlation functions [van Neerven (1986)] [Boels, Bork, Brandhuber, Engelund, Gehrmann, Gurdogan, Henn, Huber, Kazakov, Kniehl, Moch, Mooney, Naculich, Penante, Roiban, Spence, Tarasov, Travaglini, Vartanov, Wen, Yang (2010–2014)]

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Previous studies have focused on $\mathrm{tr}[\phi^{12}\phi^{12}]$ and $\mathrm{tr}[(\phi^{12})^k]$

$\rightarrow$ Study form factor of generic operator [MW(2014)]

Motivation to study form factors (2)

on-shell methods

integrability

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Form factors as bridge between on-shell methods and integrability

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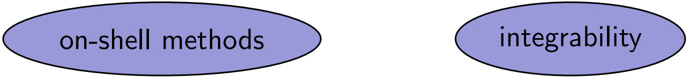

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Form factors as bridge between on-shell methods and integrability

$\rightarrow$ Revisit spectral problem via on-shell methods [MW(2014)]

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Gauge-invariant local composite operators

Gauge invariance

Traces of fields transforming covariantly under the gauge group $SU(N)$:

$$\phi_{AB},\ \psi_{ABC\alpha} = \epsilon_{ABCD}\psi^{D}_{\alpha},\ \bar{\psi}_{A\dot{\alpha}},\ F_{\mu\nu},\ \mathrm{D}_{\mu}$$

$+$ products of such traces

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Planar limit $\Rightarrow$ Sufficient to look at single-trace operators

Irreducible fields

Pauli matrices $\sigma^{\mu}_{\alpha\dot{\alpha}}$: μ, $\nu \to \alpha$, $\dot{\alpha}$

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Equation of motion, definition of field strength, Bianchi identities

Antisymmetric occurrences of α, $\dot{\alpha}$ at one field

$\rightarrow$ Several fields with totally symmetric α, $\dot{\alpha}$

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$\Rightarrow$ Irreducible fields transforming in the singleton representation $\mathcal{V}_S$ of $\mathfrak{psu}(2,2|4)$

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Picture of a spin chain

(Removed in the online version for copyright reasons)

Oscillator picture

Oscillator representation of $\mathcal{V}_S$

Bosonic oscillators: $\mathbf{a}_\alpha$, ${\mathbf{a}^\dagger}^\alpha$ and $\mathbf{b}_{\dot\alpha}$, ${\mathbf{b}^\dagger}^{\dot\alpha}$

Fermionic oscillators: $\mathbf{d}_A$, ${\mathbf{d}^\dagger}^A$

$$[\mathbf{a}_\alpha \,, {\mathbf{a}^\dagger}^\beta] = \delta_\alpha^\beta \,, \qquad [\mathbf{b}_{\dot\alpha} \,, {\mathbf{b}^\dagger}^{\dot\beta}] = \delta_{\dot\alpha}^{\dot\beta} \,, \qquad \{\mathbf{d}_A \,, \mathbf{d}^{\dagger B}\} = \delta_A^B$$

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Irreducible fields

$$\begin{aligned}
\mathrm{D}^k\,F &\;\widehat{=}\; (\mathbf{a}^\dagger)^{k+2}(\mathbf{b}^\dagger)^k \quad \mathbf{d}^{\dagger 1}\mathbf{d}^{\dagger 2}\mathbf{d}^{\dagger 3}\mathbf{d}^{\dagger 4}\,|\,0\,\rangle \\
\mathrm{D}^k\,\psi_{ABC} &\;\widehat{=}\; (\mathbf{a}^\dagger)^{k+1}(\mathbf{b}^\dagger)^k \quad \mathbf{d}^{\dagger A}\mathbf{d}^{\dagger B}\mathbf{d}^{\dagger C}\,|\,0\,\rangle \\
\mathrm{D}^k\,\phi_{AB} &\;\widehat{=}\; (\mathbf{a}^\dagger)^k \quad (\mathbf{b}^\dagger)^k \quad \mathbf{d}^{\dagger A}\mathbf{d}^{\dagger B}\,|\,0\,\rangle \\
\mathrm{D}^k\,\bar\psi_A &\;\widehat{=}\; (\mathbf{a}^\dagger)^k \quad (\mathbf{b}^\dagger)^{k+1}\mathbf{d}^{\dagger A}\,|\,0\,\rangle \\
\mathrm{D}^k\,\bar F &\;\widehat{=}\; (\mathbf{a}^\dagger)^k \quad (\mathbf{b}^\dagger)^{k+2}\,|\,0\,\rangle
\end{aligned}$$

Spin chain, dilatation operator and integrability

Dilatation operator measures (anomalous) scaling dimensions

$\rightarrow$ Observables in a CFT

[Minahan, Zarembo (2002)] [Beisert (2003)] [Beisert, Staudacher (2003)]

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$\mathfrak{su}(2)$ sector: single-trace operators built from $\uparrow = \phi_{24}$ and $\downarrow = \phi_{34}$

Heisenberg XXX spin chain $(\mathfrak{D}_2)_{i\,i+1} = 2(\mathbb{1} - \mathbb{P})_{i\,i+1}$

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Example

$\mathfrak{D}_2 \operatorname{tr}[\uparrow\uparrow\downarrow\downarrow] = 4 \operatorname{tr}[\uparrow\uparrow\downarrow\downarrow] - 4 \operatorname{tr}[\uparrow\downarrow\uparrow\downarrow]$

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Spectral problem can be solved by Bethe ansatz techniques

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Super spinor helicity variables for super form factors

Fourier transform to momentum space

$$\mathcal{F}_{\mathcal{O}}(1,\ldots,n;q) = \int \mathrm{d}^4x\, \mathrm{e}^{-iqx} \langle 1,\ldots,n|\mathcal{O}(x)|0\rangle$$

$$= \delta^4\left(q - \sum_{i=1}^{n} p_i\right) \langle 1,\ldots,n|\mathcal{O}(0)|0\rangle$$

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Nair's $\mathcal{N} = 4$ on-shell super field

$$\Phi = g^+ + \eta^A\,\bar{\psi}_A + \frac{1}{2!}\eta^A\eta^B\,\phi_{AB} + \eta^A\eta^B\eta^C\,\psi_{ABC} + \eta^1\eta^2\eta^3\eta^4\,g^-$$

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Colour-ordered super form factors

$$\mathcal{F}_{\mathcal{O}}(1,\dots,n;q) = \sum_{\sigma\in\mathbb{S}_n/\mathbb{Z}_n} \mathrm{tr}[\mathrm{T}^{a_{\sigma(1)}}\cdots\mathrm{T}^{a_{\sigma(n)}}]\hat{\mathcal{F}}_{\mathcal{O}}(\sigma(1),\dots,\sigma(n);q)$$
$$+ \text{multi-trace terms}$$

Computing form factors

Form factors can be obtained by

- BCFW and MHV recursion relations
- (generalised) unitarity
- symbols
- colour kinematic duality
- ...

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MHV amplitude [Parke, Taylor (1986)]

$$\hat{A}^{(0),\mathrm{MHV}}(1^+,\dots,i^-,\dots,j^-,\dots,n^+) = \frac{\langle ij\rangle^4\delta^4(\sum_{k=1}^n p_k)}{\langle 12\rangle\langle 23\rangle\dots\langle n1\rangle}$$

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MHV form factor [Brandhuber, Spence, Travaglini, Yang (2011)]

$$\hat{F}^{(0),\mathrm{MHV}}_{\mathrm{tr}[\phi_{12}\phi_{12}]}(1^+,\dots,i^{\phi_{12}},\dots,j^{\phi_{12}},\dots,n^+;q) = \frac{\langle ij\rangle^2 \delta^4(\sum_{k=1}^n p_k - q)}{\langle 12\rangle\langle 23\rangle\dots\langle n1\rangle}$$

Computation via Feynman rules in the free theory

$\mathsf{D}_{\alpha\dot{\alpha}}$: $\rightarrow \lambda^{\alpha} \tilde{\lambda}^{\dot{\alpha}}$

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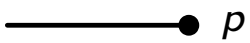
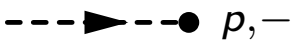
Computation via Feynman rules in the free theory

$\mathrm{D}_{\alpha\dot{\alpha}}$	:			$\rightarrow \lambda^{\alpha}\tilde{\lambda}^{\dot{\alpha}}$
ϕ_{AB}	:		$= 1$	$\rightarrow \eta^{A}\eta^{B}$
$\psi_{ABC\alpha}$	:		$= \bar{u}_{-}(p) = (\lambda^{\alpha}, 0)$	$\rightarrow \lambda^{\alpha}\eta^{A}\eta^{B}\eta^{C}$

$\mathrm{D}_{\alpha\dot{\alpha}}$	:			$\rightarrow \lambda^{\alpha}\tilde{\lambda}^{\dot{\alpha}}$
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$\psi_{ABC\alpha}$	:	 $p,-$	$= \bar{u}_{-}(p) = (\lambda^{\alpha}, 0)$	$\rightarrow \lambda^{\alpha}\eta^{A}\eta^{B}\eta^{C}$
$\bar{\psi}_{A\dot{\alpha}}$	:	 $p,+$	$= v_{+}(p) = \begin{pmatrix} 0 \\ \tilde{\lambda}^{\dot{\alpha}} \end{pmatrix}$	$\rightarrow \tilde{\lambda}^{\dot{\alpha}}\eta^{A}$

$$
\begin{array}{llll}
\mathrm{D}_{\alpha\dot{\alpha}} & : & & \to \lambda^{\alpha}\tilde{\lambda}^{\dot{\alpha}} \\
\phi_{AB} & : \quad p & = 1 & \to \eta^{A}\eta^{B} \\
\psi_{ABC\alpha} & : \quad p,- & = \bar{u}_{-}(p) = (\lambda^{\alpha}, 0) & \to \lambda^{\alpha}\eta^{A}\eta^{B}\eta^{C} \\
\bar{\psi}_{A\dot{\alpha}} & : \quad p,+ & = v_{+}(p) = \begin{pmatrix} 0 \\ \tilde{\lambda}^{\dot{\alpha}} \end{pmatrix} & \to \tilde{\lambda}^{\dot{\alpha}}\eta^{A} \\
g^{\pm} & : \quad p,\mu & = \epsilon^{\mu}_{\pm}(p, r) &
\end{array}
$$

Computation via Feynman rules in the free theory

$\mathrm{D}_{\alpha\dot{\alpha}}$	:			$\rightarrow \lambda^{\alpha}\tilde{\lambda}^{\dot{\alpha}}$
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$g^{\pm}$	:	 p,μ	$= \epsilon^{\mu}_{\pm}(p, r)$	
$F_{\alpha\beta}$	:		$\epsilon_{+} \longrightarrow 0$	

Computation via Feynman rules in the free theory

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$F_{\alpha\beta}$	:		$\epsilon_{-} \longrightarrow \lambda^{\alpha}\lambda^{\beta}$	$\rightarrow \lambda^{\alpha}\lambda^{\beta}\eta^{1}\eta^{2}\eta^{3}\eta^{4}$

$\mathrm{D}_{\alpha\dot\alpha}$	:			$\rightarrow \lambda^{\alpha}\tilde\lambda^{\dot\alpha}$
ϕ_{AB}	:	 p	$= 1$	$\rightarrow \eta^{A}\eta^{B}$
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$F_{\alpha\beta}$	:		$\epsilon_{-} \longrightarrow \lambda^{\alpha}\lambda^{\beta}$	$\rightarrow \lambda^{\alpha}\lambda^{\beta}\eta^{1}\eta^{2}\eta^{3}\eta^{4}$
$\bar F_{\dot\alpha\dot\beta}$	;		$\epsilon_{+} \longrightarrow \tilde\lambda^{\dot\alpha}\tilde\lambda^{\dot\beta}$	$\rightarrow \tilde\lambda^{\dot\alpha}\tilde\lambda^{\dot\beta}$

Computation via Feynman rules in the free theory

$\mathrm{D}_{\alpha\dot\alpha}$: $\to \lambda^{\alpha}\tilde\lambda^{\dot\alpha}$

ϕ_{AB} :  p $= 1$ $\to \eta^A\eta^B$

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$\bar\psi_{A\dot\alpha}$:  $p,+$ $= v_+(p) = \begin{pmatrix} 0 \\ \tilde\lambda^{\dot\alpha} \end{pmatrix}$ $\to \tilde\lambda^{\dot\alpha}\eta^A$

$g^{\pm}$:  p,μ $= \epsilon^{\mu}_{\pm}(p,r)$

$F_{\alpha\beta}$: $\epsilon_+ \longrightarrow 0$

$F_{\alpha\beta}$: $\epsilon_- \longrightarrow \lambda^\alpha\lambda^\beta$ $\to \lambda^\alpha\lambda^\beta\eta^1\eta^2\eta^3\eta^4$

$\bar F_{\dot\alpha\dot\beta}$; $\epsilon_+ \longrightarrow \tilde\lambda^{\dot\alpha}\tilde\lambda^{\dot\beta}$ $\to \tilde\lambda^{\dot\alpha}\tilde\lambda^{\dot\beta}$

$\bar F_{\dot\alpha\dot\beta}$: $\epsilon_- \longrightarrow 0$

Form factors as spin chains

Colour-ordered minimal super form factor for generic operator $\mathcal{O}$

$$\hat{\mathcal{F}}_{\mathcal{O}}(\Lambda_1,\dots,\Lambda_L;q) = L\delta^4\left(q-\sum_{i=1}^{L}\lambda_i^{\alpha}\tilde{\lambda}_i^{\dot{\alpha}}\right)\left(\begin{array}{c} \mathbf{a}_i^{\dagger\alpha}\to\lambda_i^{\alpha} \\ \mathbf{b}_i^{\dagger\dot{\alpha}}\to\tilde{\lambda}_i^{\dot{\alpha}} \\ \mathbf{d}_i^{\dagger A}\to\eta_i^{A} \\ \text{in oscillator picture} \end{array}\right)$$

with $\Lambda_i = (\lambda_i^{\alpha}, \tilde{\lambda}_i^{\dot{\alpha}}, \eta_i^{A})$

Minimal tree-level form factor $=$ form factor in free theory

[MW(2014)]

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Minimal tree-level form factor = form factor in free theory

Analogous replacement in algebra generators

$\rightarrow$ well-known representation on amplitudes

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Analogous replacement in algebra generators

$\rightarrow$ well-known representation on amplitudes

$\Rightarrow$ Form factors exactly implement the spin chain of $\mathcal{N}=4$ SYM theory in the language of scattering amplitudes

[MW(2014)]

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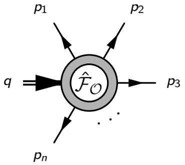
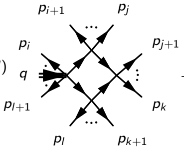
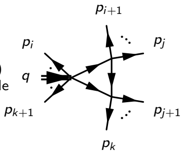
General ansatz from integral basis



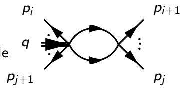
$= \sum_{i,j,k,l} c_{\text{box}}^{(i,j,k,l)}$



$+ \sum_{i,j,k} c_{\text{triangle}}^{(i,j,k)}$

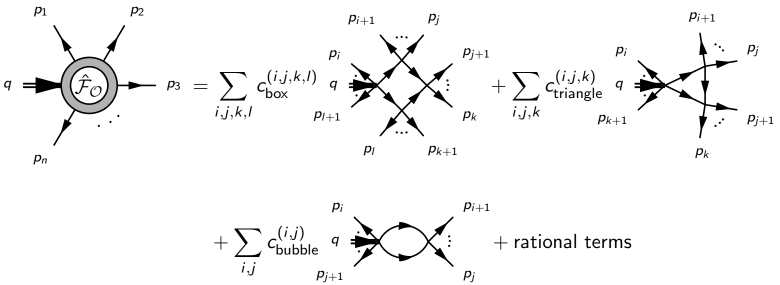


$+ \sum_{i,j} c_{\text{bubble}}^{(i,j)}$



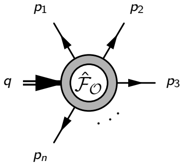
$+$ rational terms

General ansatz from integral basis

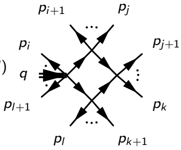


$\Rightarrow$ Sufficient to determine coefficients

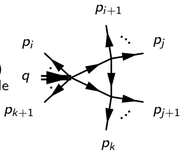
General ansatz from integral basis



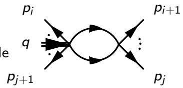
$= \sum_{i,j,k,l} c_{\text{box}}^{(i,j,k,l)}$



$+ \sum_{i,j,k} c_{\text{triangle}}^{(i,j,k)}$



$+ \sum_{i,j} c_{\text{bubble}}^{(i,j)}$



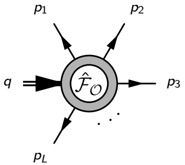
$+$ rational terms

$\Rightarrow$ Sufficient to determine coefficients

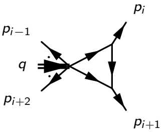
$\rightarrow$ Further simplifications for minimal form factors

Ansatz for minimal form factor

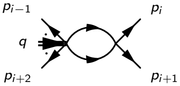
Simplified ansatz:



$= \sum_i c^{i,i+1}_{\text{triangle}}$



$+ \sum_i c^{i,i+1}_{\text{bubble}}$

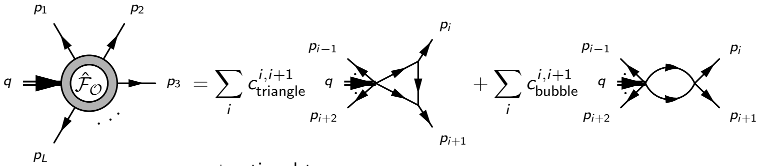


$+$ rational terms

$\Rightarrow$ Determine coefficients via cuts

Ansatz for minimal form factor

Simplified ansatz:



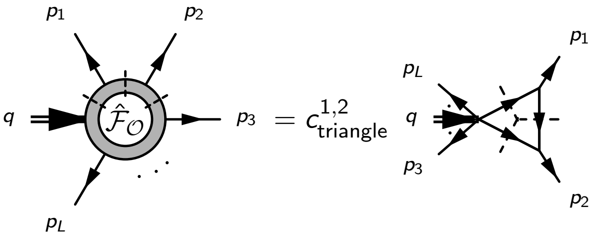
$+$ rational terms

$\Rightarrow$ Determine coefficients via cuts

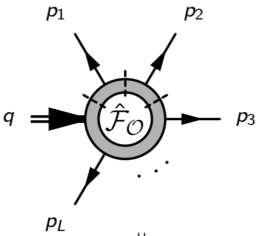
Cut: $\quad \dfrac{1}{l^2} \to \delta(l^2)\Theta(l_0)$

Triple cut and triangle coefficient

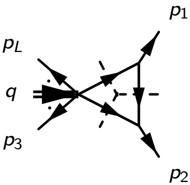
Triple cut between p_1, p_2 and the rest of the diagram:



Triple cut between p_1, p_2 and the rest of the diagram:



$= c^{1,2}_{\text{triangle}}$



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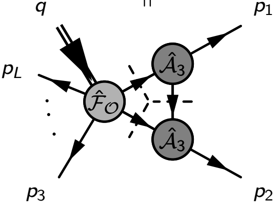
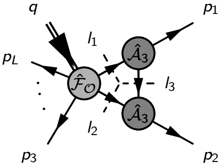
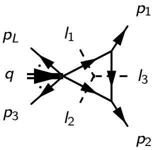


Triple cut and triangle coefficient



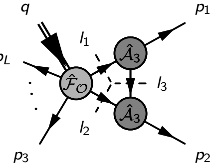
$$= C^{1,2}_{\text{triangle}}$$



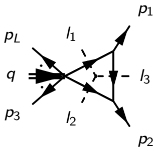
Triple cut and triangle coefficient

Integration over all intermediate degrees of freedom:

$$\int \prod_{i=1}^{3} \mathrm{d}\Lambda_{l_i}$$



$$= c^{1,2}_{\text{triangle}} \int \prod_{i=1}^{3} \mathrm{d}\Lambda_{l_i}$$

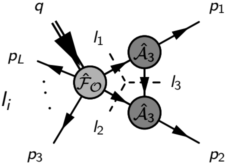


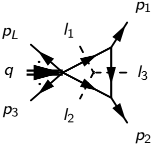
with $\mathrm{d}\Lambda_{l_i} = \mathrm{d}^2\lambda_{l_i}\, \mathrm{d}^2\tilde{\lambda}_{l_i}\, \mathrm{d}^4\eta_{l_i}$

Triple cut and triangle coefficient

The triangle coefficient:

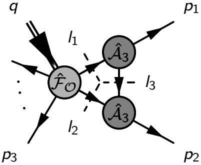
$$c^{1,2}_{\text{triangle}} = \frac{\int \prod_{i=1}^{3} \mathrm{d}\Lambda_{l_i} \; [\text{Fig. 1}]}{\int \prod_{i=1}^{3} \mathrm{d}\Lambda_{l_i} \; [\text{Fig. 2}]}$$

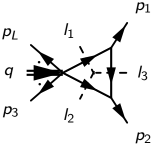




The triangle coefficient:

$$c^{1,2}_{\text{triangle}} = \frac{\int \prod_{i=1}^{3} \mathrm{d}\Lambda_{l_i} \;\; [1]}{\int \prod_{i=1}^{3} \mathrm{d}\Lambda_{l_i} \;\; [2]}$$





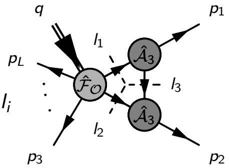
$$= -(p_1 + p_2)^2 \hat{\mathcal{F}}^{(0)}_{\mathcal{O}}(\Lambda_1, \Lambda_2, \Lambda_3, \ldots, \Lambda_L; q)$$

Triple cut and triangle coefficient

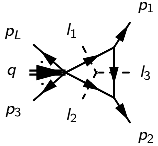
The triangle coefficient:

$$c^{1,2}_{\text{triangle}} =$$

$$\int \prod_{i=1}^{3} \mathrm{d}\Lambda_{l_i}$$



$$\int \prod_{i=1}^{3} \mathrm{d}\Lambda_{l_i}$$

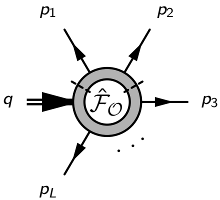


$$= -(p_1 + p_2)^2 \hat{\mathcal{F}}^{(0)}_{\mathcal{O}}(\Lambda_1, \Lambda_2, \Lambda_3, \ldots, \Lambda_L; q)$$

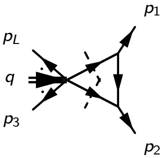
$\Rightarrow$ Universal for all operators

Double cut and bubble coefficient

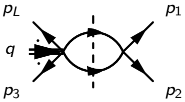
Double cut between p_1, p_2 and the rest of the diagram:



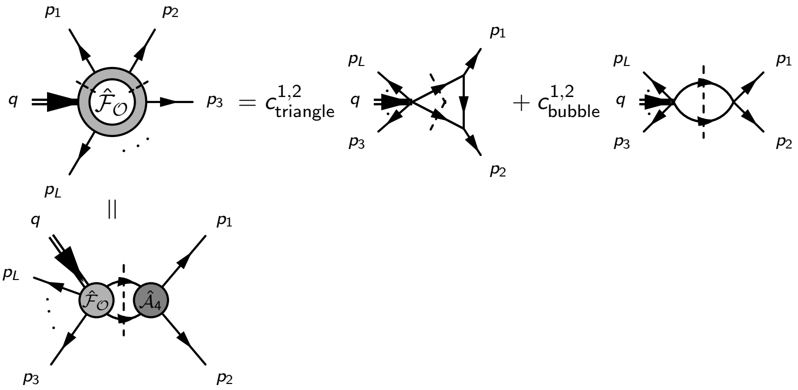
$= c^{1,2}_{\text{triangle}}$



$+ c^{1,2}_{\text{bubble}}$



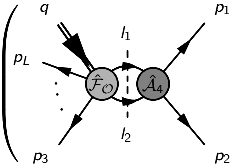
Double cut between p_1, p_2 and the rest of the diagram:



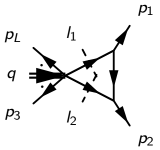
The bubble coefficient:

$c^{1,2}_{\text{bubble}} =$

$\int \prod_{i=1}^{2} \mathrm{d}\Lambda_{l_i} \Bigg($

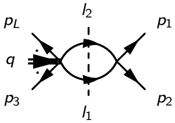


$- c^{1,2}_{\text{triangle}}$



$\Bigg)$

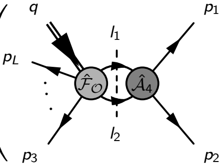
$\int \prod_{i=1}^{2} \mathrm{d}\Lambda_{l_i}$

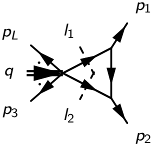


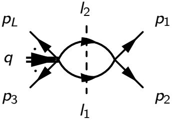
Double cut and bubble coefficient

The bubble coefficient:

$$c^{1,2}_{\text{bubble}} = \frac{\displaystyle\int \prod_{i=1}^{2} \mathrm{d}\Lambda_{l_i} \left(\text{[diagram 1]} - c^{1,2}_{\text{triangle}}\ \text{[diagram 2]} \right)}{\displaystyle\int \prod_{i=1}^{2} \mathrm{d}\Lambda_{l_i}\ \text{[diagram 3]}}$$







$$= B_{1\,2}\, \hat{\mathcal{F}}^{(0)}_{\mathcal{O}}(\Lambda_1, \Lambda_2, \Lambda_3, \dots, \Lambda_L; q)$$

Bubble coefficient operator

$$B_{i\,i+1}\hat{\mathcal{F}}^{(0)}_{\mathcal{O}}(\Lambda_1,\dots,\Lambda_L;q) = -2\delta_{C_i,0}\int_0^{\pi/2}\mathrm{d}\theta\cot\theta\Big(\hat{\mathcal{F}}^{(0)}_{\mathcal{O}}(\Lambda_1,\dots,\Lambda_i,\Lambda_{i+1},\dots,\Lambda_L;q) - \hat{\mathcal{F}}^{(0)}_{\mathcal{O}}(\Lambda_1,\dots,\Lambda'_i,\Lambda'_{i+1},\dots,\Lambda_L;q)\Big)$$

with

$$\begin{pmatrix}\Lambda'_i\\ \Lambda'_{i+1}\end{pmatrix} = \begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\Lambda_i\\ \Lambda_{i+1}\end{pmatrix}, \quad \Lambda_i = (\lambda_i^\alpha,\tilde{\lambda}_i^{\dot{\alpha}},\eta_I^A)$$

Bubble coefficient operator

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Polynomial in $\cos\theta$ and $\sin\theta$

$\Rightarrow$ Evaluates to Euler β-function or harmonic number

Bubble coefficient operator

$$B_{i\,i+1}\hat{\mathcal{F}}^{(0)}_{\mathcal{O}}(\Lambda_1,\dots,\Lambda_L;q) =$$
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$$-\hat{\mathcal{F}}^{(0)}_{\mathcal{O}}(\Lambda_1,\dots,\Lambda'_i,\Lambda'_{i+1},\dots,\Lambda_L;q)\Big)$$

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Polynomial in $\cos\theta$ and $\sin\theta$

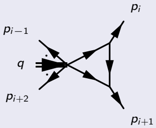
$\Rightarrow$ Evaluates to Euler β-function or harmonic number

Operator $\mathcal{O}$ in $\mathfrak{su}(2)$ sector $\Rightarrow$ $B_{i\,i+1} = -(\mathbb{1}-\mathbb{P})_{i\,i+1}$

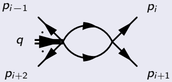
Summary

One-loop minimal form factor of a generic operator $\mathcal{O}$

$$\hat{\mathcal{F}}_{\mathcal{O}}^{(1)}(1,\dots,L;q) = -\sum_{i=1}^{L}(p_i+p_{i+1})^2\hat{\mathcal{F}}_{\mathcal{O}}^{(0)}(1,\dots,L;q)$$



$$+\sum_{i=1}^{L}B_{i\,i+1}\hat{\mathcal{F}}_{\mathcal{O}}^{(0)}(1,\dots,L;q)$$



$+$rational terms

[MW(2014)]

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IR divergences

IR divergence from triangle integral (universal)

$$\left.\frac{\hat{\mathcal{F}}^{(1)}_{\mathcal{O}}(1,\dots,L;q)}{\hat{\mathcal{F}}^{(0)}_{\mathcal{O}}(1,\dots,L;q)}\right|_{\mathrm{IR}} = -\frac{1}{\varepsilon^2}\sum_{i=1}^{L}(-s_{i\,i+1})^{-\varepsilon}\,, \quad D = 4-2\varepsilon$$

IR divergences

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Agrees with universal BDS-ansatz-type form

[Bern, Dixon, Smirnov (2005)]

$$\log\left(\frac{\hat{\mathcal{F}}_{\mathcal{O}}(1,\dots,n;q)}{\hat{\mathcal{F}}^{(0)}_{\mathcal{O}}(1,\dots,n;q)}\right)$$
$$= \sum_{l=1}^{\infty} g^{2l}\left[-\frac{\gamma^{(l)}_{\text{cusp}}}{8(l\varepsilon)^2} - \frac{\mathcal{G}^{(l)}_0}{4l\varepsilon}\right]\sum_{i=1}^{n}(-s_{i\,i+1})^{-l\varepsilon} + \text{Fin}(g^2) + \mathcal{O}(\varepsilon)$$

(Form checked for BPS form factors in [Brandhuber, Spence, Travaglini, Yang (2010)], [Gehrmann, Henn, Huber (2011)], [Brandhuber, Travaglini, Yang (2012)], [Brandhuber, Penante, Travaglini, Wen (2014)])

UV divergences

Operator renormalisation

$$\mathcal{O}^a_{\text{ren}} = \mathcal{Z}^a{}_b \mathcal{O}^b_{\text{bare}}\,, \qquad \mathcal{Z}^a{}_b = \delta^a{}_b + g^2 (\mathcal{Z}^{(1)})^a{}_b + \mathcal{O}(g^3)$$

UV divergences

Operator renormalisation

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Renormalised form factor

$$\hat{\mathcal{F}}_{\mathcal{O}^a_{\text{ren}}}(1, \ldots, L; q) = \mathcal{Z}^a{}_b \hat{\mathcal{F}}_{\mathcal{O}^b_{\text{bare}}}(1, \ldots, L; q)$$

UV divergences

Operator renormalisation

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Renormalised form factor

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UV-finiteness of renormalised form factor $\Rightarrow$

$$(\mathcal{Z}^{(1)})^a{}_b \hat{\mathcal{F}}^{(0)}_{\mathcal{O}^b_{\text{bare}}}(1,\dots,L;q) = -\,\hat{\mathcal{F}}^{(1)}_{\mathcal{O}^a_{\text{bare}}}(1,\dots,L;q)\Big|_{\text{UV}}$$

$$= -\Big(\sum_{i=1}^{L} B_{i\,i+1}\Big)^a{}_b \hat{\mathcal{F}}^{(0)}_{\mathcal{O}^b_{\text{bare}}}(1,\dots,L;q)\frac{1}{\varepsilon}$$

Dilatation operator

Anomalous part of dilatation operator

$$\delta\mathfrak{D} = \lim_{\varepsilon\to 0} \varepsilon g \frac{\partial}{\partial g} \ln \mathcal{Z}$$

Dilatation operator

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Complete one-loop dilatation operator

$$\mathfrak{D}_2 = \frac{1}{g^2} \lim_{\varepsilon\to 0} \varepsilon g \frac{\partial}{\partial g} \left(g^2 \mathcal{Z}^{(1)} \right) = -2 \sum_{i=1}^{L} B_{i\,i+1}$$

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One-loop dilatation operator density

$$(\mathfrak{D}_2)_{i\,i+1} = -2 B_{i\,i+1}$$

Dilatation operator

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One-loop dilatation operator density

$$(\mathfrak{D}_2)_{i\,i+1} = -2B_{i\,i+1}$$

$\Rightarrow$ Agrees with result of [Beisert (2003)] in formulation of [Zwiebel (2007)] after replacing oscillators by super spinor helicity variables. (Proof of a connection between amplitudes and dilatation operator which was observed in [Zwiebel (2011)].)

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Konishi operator

- Prime example of non-protected operators
- $\mathcal{K} = \mathrm{tr}[\phi^I \phi^I]$

Konishi operator

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  $$\mathcal{K}_{\text{ren}} = \mathcal{Z}\mathcal{K}_{\text{bare}} \text{ with } \mathcal{Z} = \exp\left[\sum_{\ell=1}^{\infty} \frac{g^{2\ell}\gamma_{\mathcal{K}}^{(\ell)}}{2\ell\varepsilon}\right]$$

- Anomalous dimension $\gamma_{\mathcal{K}}$ known via field theory up to five loops [Eden, Heslop, Korchemsky, Smirnov, Sokatchev (2012)] and via integrability up to ten loops [Marboe, Volin (2014)]

Konishi operator

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Konishi operator

- Prime example of non-protected operators
- $\mathcal{K} = \text{tr}[\phi^I \phi^I]$
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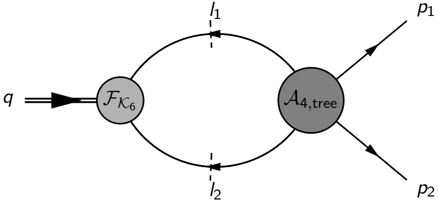
- $\mathcal{K}_6 \neq \mathcal{K}$ unless $D = 4$

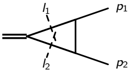
One-loop Konishi form factor

Planar double cut

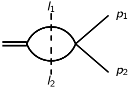


$$f^{(1)}_{\mathcal{K}_6,(\phi,\phi)}\Big|_{q^2} = \frac{F^{(1)}_{\mathcal{K}_6,(\phi,\phi)}}{F^{(0)}_{\mathcal{K}_6,(\phi,\phi)}}\Bigg|_{q^2} = \cdots = \int \mathrm{dPS}_{l_1,l_2}\left(\frac{\langle l_1 l_2\rangle\langle 12\rangle}{\langle 1 l_1\rangle\langle 2 l_2\rangle} + 6\,\frac{\langle l_1 2\rangle\langle l_2 1\rangle}{\langle 12\rangle\langle l_1 l_2\rangle}\right)$$

$$= -s_{12}$$



$$+\,6\,\frac{(l_1+p_2)^2}{s_{12}}$$



[Nandan, Sieg, MW, Yang (2014)]

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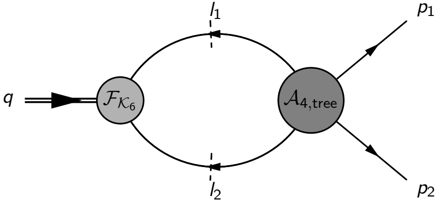

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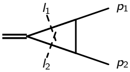
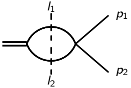
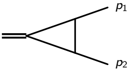
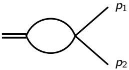
$= -s_{12}$  $+\,6\,\dfrac{(l_1+p_2)^2}{s_{12}}$ 

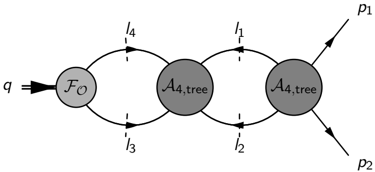
Lift and Passarino-Veltman reduction

$f^{(1)}_{\mathcal{K}_6,(\phi,\phi)} = 2\Big(-s_{12}$  $-\,3$  $\Big)$

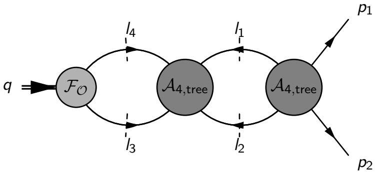
[Nandan, Sieg, MW, Yang (2014)]

Two-point Konishi form factor

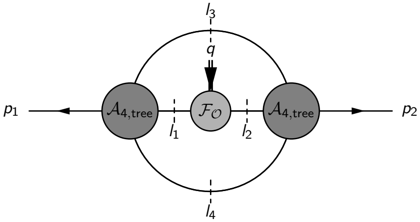
Planar double-double cut



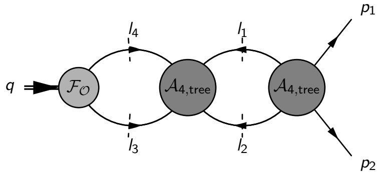
Planar double-double cut



Non-planar double-double cut



Planar double-double cut



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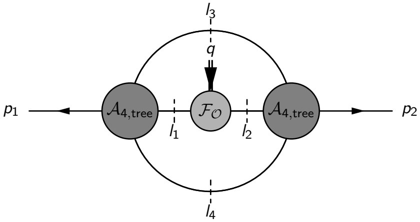
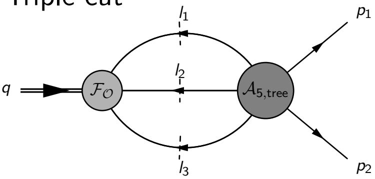
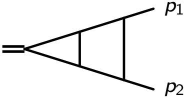


Triple cut

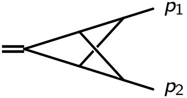


Final result:

$$f^{(2)}_{\mathcal{K}_6,2} = \underbrace{s_{12}^2 \left(4 \right.}_{f^{(2)}_{\mathrm{BPS},2}}$$

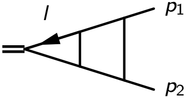


$+$



$\Big)$

$$- 6(l+p_1)^2(l+p_2)^2 \Big(4$$



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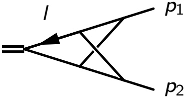
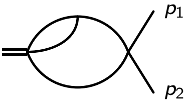


$\Big)$

$$+ \frac{36}{s_{12}}$$



[Nandan, Sieg, MW, Yang (2014)]

Subtleties (1)

Divergences $\Rightarrow$ Regularise in $D = 4 - 2\varepsilon$ dimensions

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Gauge field A_M, $M = 1, \ldots, 10$

$$\rightarrow \quad \begin{array}{ll} A_\mu, & \mu = 1, \ldots, D = 4 - 2\varepsilon \\ \phi_I, & I = 1, \ldots, 10 - D = 6 + 2\varepsilon \end{array}$$

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Metric $\eta_{MN} = \eta_{\mu\nu} - \delta_{IJ}$

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Feynman diagrams

Vector index loop: $\eta_{\mu\nu}\eta^{\mu\nu} = D = 4 - 2\varepsilon$
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Dimensional reduction

$\rightarrow$ gluon + scalar

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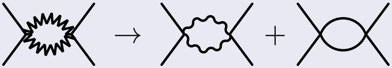
$\Rightarrow$ independent of ε



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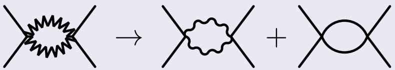

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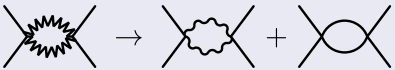

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E.g. $\mathcal{K} = \delta^{IJ}\,\mathrm{tr}[\phi_I \phi_J]$

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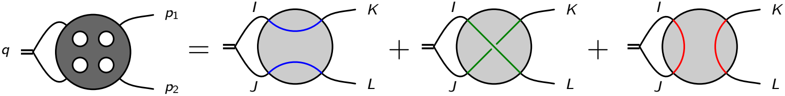
Generic multi-loop diagram with incoming operator $\mathcal{O} = \mathrm{tr}[\phi_I \phi_J]$ and outgoing scalar fields ϕ_K and ϕ_L. R-charge conservation:



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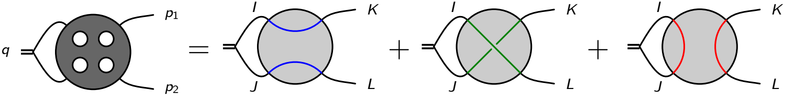

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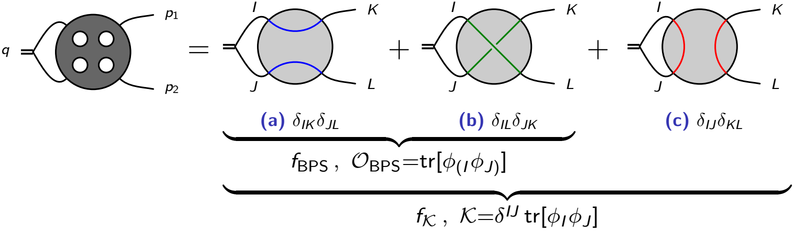

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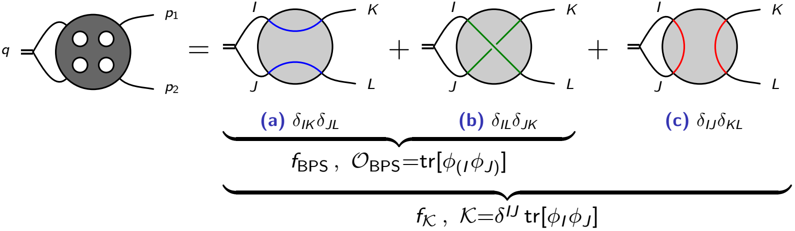

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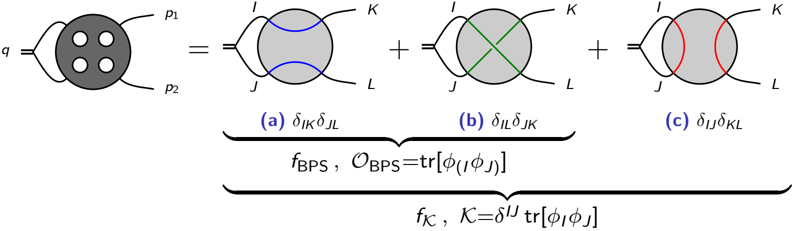

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$\rightarrow$ Should have $\delta^{IJ}\delta_{IJ} = N_\phi = 6 + 2\varepsilon$ instead of $N_\phi = 6$

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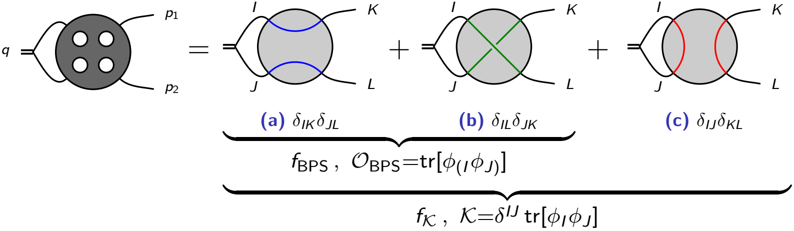

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Lift intermediate states:

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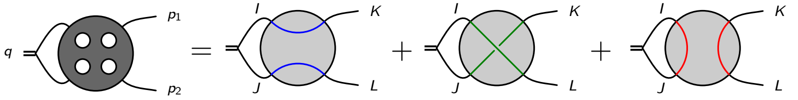

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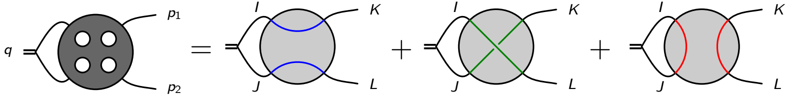

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$\Rightarrow$ New kind of rational term at $\ell = 1$, $\frac{1}{\varepsilon}$-pole altered at $\ell = 2$

[Nandan, Sieg, MW, Yang (2014)]

Anomalous dimension

Universal IR structure

$$(\log f_{\mathcal{K},\mathrm{ren}})^{(2)} = \left(f^{(2)}_{\mathcal{K},\mathrm{bare}} + \mathcal{Z}^{(1)}_{\mathcal{K}} f^{(1)}_{\mathcal{K},\mathrm{bare}} + \mathcal{Z}^{(2)}_{\mathcal{K}}\right) - \frac{1}{2}\left(f^{(1)}_{\mathcal{K},\mathrm{bare}} + \mathcal{Z}^{(1)}_{\mathcal{K}}\right)^2$$

[Nandan, Sieg, MW, Yang (2014)]

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$$
\Rightarrow \ \mathcal{Z}^{(2)}_{\mathcal{K}} = \frac{18}{\varepsilon^2} - \frac{12}{\varepsilon} = \frac{(\gamma^{(1)}_{\mathcal{K}})^2}{8\varepsilon^2} + \frac{\gamma^{(2)}_{\mathcal{K}}}{4\varepsilon}
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$$\Rightarrow \quad \gamma^{(1)}_{\mathcal{K}} = 12 \text{ and } \gamma^{(2)}_{\mathcal{K}} = -48$$

[Nandan, Sieg, MW, Yang (2014)]

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$\Rightarrow \quad \gamma^{(1)}_{\mathcal{K}} = 12$ and $\gamma^{(2)}_{\mathcal{K}} = -48$

Agrees with results of [Anselmi, Grisaru, Johansen (1997)], [Anselmi, Freedman, Grisaru, Johansen (1997)] and [Bianchi, Kovacs, Rossi, Stanev (1999,2000)],[Eden, Schubert, Sokatchev (2000)]

[Nandan, Sieg, MW, Yang (2014)]

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Thank you!