B0] The anatomy of one-loop amplitudes in pure spinor superspace

based on: C. Mafra, OS 1404.4986, 1408.3605

B1]

$\boxed{\text{scattering amplitudes}}$

$\Uparrow$ IV

$\boxed{\text{BRST (pseudo-) invariants}}$

[~~to~~ appear] $\Uparrow$ III

$\boxed{\text{Berends Giele currents}}$

[1408.3605] $\Uparrow$ II

$\boxed{\text{multiparticle superfields}}$

[1404.4986] $\Uparrow$ I

$\boxed{\text{10 d } \mathcal{N}=1 \text{ SYM superfields}}$

[0405072, 0603165] $\Uparrow$

$\boxed{\text{gluon \& gluino pol's } e^m, \chi^\alpha}$

bosonic pure spinor

$$\lambda^\alpha \gamma^m_{\alpha\beta} \lambda^\beta = 0$$

p.s. superspace $\{x^m, \theta^\alpha, \lambda^\alpha\}$

$\langle \lambda^3 \theta^5 \rangle = 1$ automated [1007, 4999]

$$Q = \lambda^\alpha D_\alpha = \lambda^\alpha \left(\frac{\partial}{\partial \theta^\alpha} + \frac{k^m}{2} (\gamma_m \theta)_\alpha \right)$$

$$A_\alpha(x,\theta) = e^{ikx} \left\{ \frac{1}{2} (\gamma^m \theta)_\alpha e_m - \frac{1}{3} (\chi \gamma_m \theta)(\gamma^m \theta)_\alpha + \theta^3 e k + \theta^4 \chi k + \ldots \right\}$$

B2] I Multiparticle superfields

(un-) integrated vertex operators $\qquad$ θ-expansion

$$V_1 = \lambda^\alpha A^1_\alpha \, , \qquad U_1 = \partial\theta^\alpha A^1_\alpha + \Pi^m A^1_m + d_\alpha W_1^\alpha + \frac{1}{2} N_{mn} F_1^{mn}$$

$h=1$ "bookkeeping var's"

BRST invariance

$$Q V_{1(2)} = 0 \; {}^{+ s_{12} V_1 V_2} \, , \qquad Q \int U_{1(2)} = \int \partial V_{1(2)} \; {}^{+ s_{12}(V_1 U_2 - V_2 U_1)} = 0 \iff \text{e.o.m.}$$

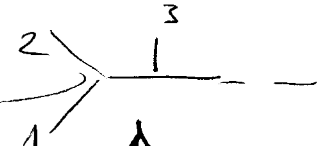
$\Downarrow$ $k_m e^m = k \chi = 0$

$$\begin{cases} 2 D_{(\alpha} A^{1(2)}_{\beta)} = \gamma^m_{\alpha\beta} A^{1(2)}_m & + s_{12} (A^1_\alpha A^2_\beta - A^2_\alpha A^1_\beta) \\ D_\alpha A^m_{1(2)} = (\gamma^m W_{1(2)})_\alpha + k^m_{1(2)} A^{1(2)}_\alpha & + s_{12} (A^1_\alpha A^m_2 - A^2_\alpha A^m_1) \\ D_\alpha W^\beta_{1(2)} = \frac{1}{4} (\gamma^{mn})_\alpha{}^\beta F^{1(2)}_{mn} & + s_{12} (A^1_\alpha W^\beta_2 - A^2_\alpha W^\beta_1) \end{cases}$$

B3| Define multiparticle fields by OPE

$$U_1(z_1)\, U_2(z_2) \sim (z_1-z_2)^{\alpha' k_1\cdot k_2 -1} \Big[\underbrace{\partial\theta^\alpha A_\alpha^{12} + \Pi_m A_{12}^m + d_\alpha W_{12}^\alpha + \tfrac{1}{2} N^{mn} F_{12}^{mn}}_{=: U_{12}} + \partial_1(\_\_) + \partial_2(\_\_)$$

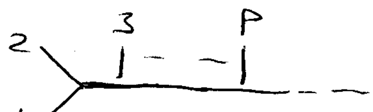
e.g. $\underset{=\lambda^\alpha A^{12}_\alpha}{V_{12}} = \frac{1}{2}\Big[(k_2\cdot A_1)V_2 + A_2^m(\gamma_m W_1)_\alpha - (1\leftrightarrow 2)\Big]$

More particles by recursion with 

$$Q\,\hat V_{123} = (\cancel{s_{123}}-\cancel{s_{12}})\, V_{12}V_3 + \cancel{s_{12}}\,(V_1V_{23} - V_2V_{13})$$

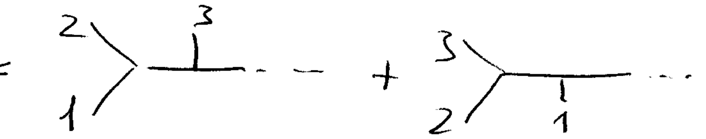
$\Rightarrow Q\,\hat V_{[123]} \Rightarrow$ remove BRST-exact cpt.

$V_{123} := \hat V_{123} - \hat V_{[123]} \iff$ Jacobi id. $f^{a[12}f^{3]ab}=0$

$V_B, A_B^m, W_B^\alpha, F_B^{mn} \iff$  $\iff f^{12a}f^{a3b}\ldots f^{ypz}$
with $B = 12\ldots p$

B4| II ~~BRST-invariants~~ Berends–Giele currents $\Rightarrow$ Q & eom nicer

~~(i) Berends-Giele currents~~

$M_{123} =$  $= \dfrac{V_{123}}{s_{12}s_{123}} + \dfrac{V_{321}}{s_{23}s_{123}}$ = ~~off-shell~~ 4pt tree

reward: $QM_{123} = M_{12}M_3 + M_1M_{23}$ $\quad \dfrac{V_{23}}{s_{23}} \quad V_1$

deconcatenation: $QM_{123\ldots p} = \sum_{j=1}^{p-1} M_{12\ldots j}M_{j+1\ldots p} = \sum_{XY=12\ldots p} M_X M_Y$
$(p+1)$ leg tree

example: n-pt tree [1012.3981] $\quad \Big\langle \sum_{j=1}^{n-2} M_{12\ldots j}M_{j+1\ldots n-1}M_n \Big\rangle$

guided by string prescript.: $\langle V_1 \underbrace{U_2 \ldots U_{n-2}}_{n-3} V_{n-1}V_n\rangle$

B5)

Likewise : BG current $(A^m_B, \lambda^\alpha_B, F^{mn}_B) \leftrightarrow (A^m_B, W^\alpha_B, F^{mn}_B)$

$$Q A^m_B = (\lambda\gamma^m W_B) + k^m_B M_B + \sum_{XY=B} (M_X A^m_Y - M_Y A^m_X)$$

$$Q W^\alpha_B = \frac{1}{4}(\lambda\gamma^{mn})^\alpha F^{mn}_B + \sum_{XY=B} (M_X W^\alpha_Y - M_Y W^\alpha_X)$$

B6)

(III) BRST invariants @ 1-loop

string prescription : $\langle (Nb) V_1 U_2 \ldots U_n \rangle$ → need zero mode $d_\alpha d_\beta N^{mn}$

regulation & b-ghost $\Leftrightarrow W^\alpha_A W^\beta_B F^{mn}_C$

unique $\mathcal{O}(\lambda^2)$ tensor : $M_{A,B,C} = \frac{1}{3}(\lambda\gamma_m W_A)(\lambda\gamma_n W_B) F^{mn}_C + (A \leftrightarrow B,C)$

$$0 = (\lambda\gamma_m\lambda) = (\lambda\gamma_{mnp}\lambda) \Leftarrow (\lambda\gamma_n)_\alpha(\lambda\gamma^n)_\beta$$

$$Q M_{A,B,C} = \sum_{XY=A} (M_X M_{Y,B,C} - M_Y M_{X,B,C}) + (A \leftrightarrow B,C)$$

4pt box numerator : $C_{1|2,3,4} = M_1 M_{2,3,4}$, Q-closed

5pt box numerator : $C_{1|23,4,5} = M_1 M_{23,4,5} + \underbrace{M_{12} M_{3,4,5}}_{M_1 \otimes M_2 M_{3,4,5}} - M_{13} M_{2,4,5}$

∃ recursion for $C_{1|A,B,C} = M_1 M_{A,B,C}$ + BRST-compl. with $M_A \otimes M_B = M_{AB}$

B7| rank r tensors $\leftrightarrow$ zero modes $(\Pi)^r d_\alpha d_\beta N^{mn}$

$$M^m_{A,B,C,D} = \left[M_{A,B,C} A^m_D + (D \leftrightarrow C,B,A) \right] + W^m_{A,B,C,D}$$

compensate for $QA^m_B \supset (\lambda\gamma^m W_B)$

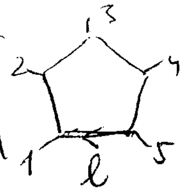
$$Q M^m_{A,B,C,D} = k^m_A M_A M_{B,C,D} + \sum_{XY=A} (M_X M^m_{Y,B,C,D} - (X \leftrightarrow Y)) + (A \leftrightarrow B,C,D)$$

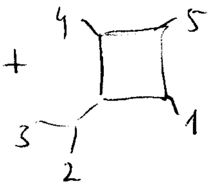
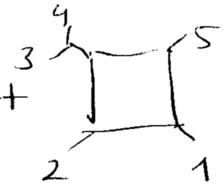
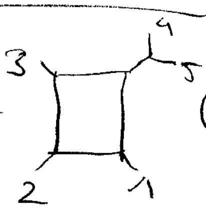
5pt vector pentagon $C^m_{1|2,3,4,5} = M_1 M^m_{2,3,4,5} + \left[\underbrace{M_{12} k^m_2 M_{3,4,5}} + (2 \leftrightarrow 3,4,5) \right]$

$M_1 \otimes k^m_2 C_{2|3,4,5}$

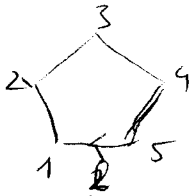
higher rank : $M^{m_1 \ldots m_r}_{B_1,B_2 \ldots B_{r+3}} = (A)^r M_{B,C,D} + (A)^{r-1} W^m_{A,B,C,D}$

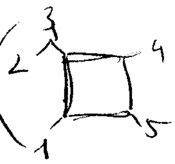
B8 | IV 5pt SYM & SUGRA, open & closed strings

$A(\underbrace{1,2,3,4,5}_{\text{single-trace}}) = \int d^{10}\ell \Big\{$  $\underbrace{\tfrac{1}{2}\Big[2\ell_m C^m_{1|2,3,4,5} + \{ s_{23} C_{1|23,4,5} + (23 \leftrightarrow 24, 25, 34, 35, 45) \} \Big]}$

$+$  $C_{1|23,4,5} +$  $C_{1|34,2,5} +$  $C_{1|45,2,3} \Big\} \quad N_{1|2,3,4,5}(\ell)$

"square all numerators"

$M_5 = \int d^{10}\ell \Big\{ \Big[$  $|N_{1|2,3,4,5}(\ell)|^2 + \text{perm}(2,3,4,5) \Big]$

$+ \Big[\Big($  $+ \text{perm}(23,4,5) \Big) \, s_{23} \, |C_{1|23,4,5}|^2 + (23 \leftrightarrow 24, 25, 34, 35, 45) \Big] \Big\}$

B9 | $A^{\text{open}}(1,2,3,4,5) = \int_0^\infty \frac{dt}{t} \int dz_2 \ldots dz_5 \prod_{i<j} e^{\alpha' k_i k_j G_{ij}} \Big\{ \underbrace{\partial G_{23} s_{23} C_{1|23,4,5} + \binom{23 \to}{24 \ldots 45}}_{=: K_5^{\text{open}}}$

$$M_5^{\text{closed}} = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^5} \int d^2 z_2 \ldots d^2 z_5 \prod_{i<j} e^{\frac{\alpha'}{2} k_i k_j G_{ij}} \Big\{ |K_5^{\text{open}}|^2 + \frac{\pi}{\tau_2} C^m_{1|2,3,4,5} \tilde{C}^m_{1|2 \ldots 5} \Big\}$$