Aspects of Higher Spin Symmetries in Various Dimensions

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In progress with Murat Günaydin, Evgeny Skvortsov & Massimo Taronna

Overture

Motivations

- Why study higher spin symmetries?

Maximal symmetry

HS symmetries can not result from breaking of higher symmetries

$\Downarrow$

Manifest at any scale including Planck scale and above $\Rightarrow$ HS gauge theory captures quantum gravity effects

[Fradkin-Vasiliev]

Gravity is sourced by HS fields & vice versa $\Rightarrow$ Einstein gravity cannot be obtained as a truncation & Riemannian geometry is not an appropriate tool

One loop calculations of free energy show that HS theories are one-loop UV finite theories of quantum gravity

[Giombi-Klebanov '13, Giombi-Klebanov-Safdi '14]

Stringy motivations

There are large N free CFTs with conserved HS currents in d=4 (e.g. N=4 SYM in the zero ‘t Hooft coupling limit)

↕ AdS/CFT

Tensionless limit of type IIB string theory in AdS_5 x S^5 must reduce to a HS gauge theory (coupled to infinite tower of massless fields).

[Ferrara-Fronsdal ’98,Haagi-Mani-Sundborg ’00, Konstein-Vasiliev-Zaikin ’00, Witten ’01, Beisert-Bianchi-Morales-Samtleben ’04, Sagnotti ’11, Jevicki et al, Douglas et al]

Symmetry ⇒ Solvability

∞ dimensional conformal (Virasoro) symmetry in two dimensions gives a host of exactly solvable models

[Belavin-Polyakov-Zamolodchikov '84]

Conformal symmetry finite dimensional in d>2, so we need more symmetries → ∞ HS symmetries!

However, exact conformal HS symmetry is too restrictive & gives free field theories but approximate HS symmetry gives non-trivial interacting CFTs

[Maldacena-Zhiboedov '11]

Motivations

- Why study higher spin symmetries?
- Higher spin holography.

HS/Vector model holography

HS/Vector model duality (AdS_4/CFT_3) conjectures
[Klebanov-Polyakov, Sezgin-Sundell, Leigh-Petkou '02, Giombi-Yin '09,'10]

AdS_4 HS theories are dual to free or critical O(N) boson/ fermion vector model depending on boundary conditions

Generic boundary conditions (parity violating) break some HS symmetry and correspond to non-linear boundary conformal theories where HS currents interact with HS gauge fields and acquire anomalous dimensions [Vasiliev '12]

Vasiliev fields as string bits

Vasiliev theory with U(M) Chan-Paton factors & $\mathscr{N}$=6 b.c.

Large N,k

Finite M

$\theta_0 = \pi\lambda/2$

$U(N)_k \times U(M)_{-k}$ ABJ theory

Strong bulk 't Hooft coupling

$\lambda_{bulk} = M/N$

$R_{AdS}/\ell_{string} = (\lambda)^{1/4}$

$\int_{CP^1} B = (N-M)/k$

Type IIA string theory in $AdS_4 \times CP^3$

Bound states of HS particles ············► Strings

[Giombi-Minwalla-Prakash-Trivedi-Wadia '11, Chang-Minwalla-Sharma-Yin '12]

AdS_3/CFT_2 Story - Minimal model Holography

[Gaberdiel-Gopakumar '11]: The role of vector models in 2d is played by $W_{N,k}$ minimal models.

Conjecture: Holographic duals are AdS_3 Vasiliev higher spin theory (tower of ∞ HS fields coupled to massive scalars).

Vasiliev higher spin symmetry organizes all the states of the $(T^4)^{N+1}/S_{N+1}$ orbifold symmetric product CFT = Tensionless limit of strings on $AdS_3 \times S^3 \times T^4$

[Gopakumar Strings '14, Gaberdiel-Gopakumar '14]

Motivations

- Why study higher spin symmetries?
- Higher spin holography.
- Extension to d=4 and d=6?

Problems

Conformal HS algebras in d=3 are conveniently described in terms of Lorentz covariant twistorial oscillators due to $SO(3,2) \approx Sp(4,R)$

Extension to d = 4 and 6 is not straightforward as the algebras described by Lorentz covariant twistorial oscillators are

d = 4: $Sp(8,R) \supset SU(2,2) \approx SO(4,2)$

d = 6: $Sp(16,R) \supset SO^*(8) \approx SO(6,2)$

Punchline

The conformal HS algebras in 4 and 6 dimensions are naturally formulated in terms of *nonlinear* twistors which transform *nonlinearly* under the respective Lorentz groups.

These algebras admit one parameter continuous (4d) and discrete (6d) deformations which describe mixed symmetry HS algebras.

Straightforward supersymmetric extensions.

Prelude

HS algebras and singleton/doubleton representations

SO(d,2)

$\swarrow \qquad \searrow$

Conformal group in d>2 space-time dimensions

AdS$_{d+1}$ symmetry group

Positive energy unitary representations or lowest weight representations

Modules labeled by compact subgroup SO(d)xSO(2)

Unitarity = Hilbert space with positive definite norm

SO(2) label (conformal dimension or AdS energy)

Bounds on labels $\Delta \geq f(SO(D)\ \text{labels})$

Saturate the bounds

Various "short" and "semi-short" representations

Notion of masslessness is tricky to define in AdS

Gauge degrees of freedom or reduced d.o.f.

SO(4,2)

Positive energy UIR's classified [Mack '77]

	Labels	4d Poincare content	AdS_5 content
Short/singleton/ doubleton (protected)	$j_1 j_2 = 0$, $\Delta = j_1+j_2+1$	$m = 0$ helicity $= j_1 - j_2$	No Poincare limit, live on the boundary [Günaydin-Marcus '84]
Semi-short	$j_1 \neq 0$, $j_2 \neq 0$ $\Delta = j_1+j_2+2$	$m > 0$ spin $= j_1 + j_2$	Massless symmetric + mixed
Chiral semi-short	$j_1 j_2 = 0$, $\Delta > j_1+j_2+1$	$m > 0$ spin $= j_1 + j_2$	Chiral massless
Long	$j_1 \neq 0$, $j_2 \neq 0$ $\Delta > j_1+j_2+2$	$m > 0$, $s = \|j_1 - j_2\|,\dots, j_1 + j_2$	Massive

Oscillator methods for SO(4,2) ≈ SU(2,2)

[Günaydin-Marcus, Günaydin-Nieuwenhuizen-Warner ’85, Günaydin-Minic-Zagermann ’98,…]

$$[a_i(\xi)\,,\, a^j(\eta)] = \delta_i{}^j\, \delta_{\xi,\eta}\,,\ [b_r(\xi)\,,\, b^s(\eta)] = \delta_r{}^s\, \delta_{\xi,\eta}$$

$$i,j = 1,2 \quad r,s = 1,2 \quad \xi, \eta = 1,2,\ldots,P$$

generations of oscillators

Lowering operators $L_{ir} = a_i \,.\, b_r$

Raising operators $L^{ir} = a^i \,.\, b^r$

Maximal compact subgroup K of SU(2,2) = $SU(2)_L \times SU(2)_R \times U(1)_E$

$SU(2)_L$	$L_i{}^j = a^j \,.\, a_i - \frac{1}{2}\, \delta_i{}^j\, a^k \,.\, a_k$
$SU(2)_R$	$R_i{}^j = b^j \,.\, b_i - \frac{1}{2}\, \delta_i{}^j\, b^k \,.\, b_k$
AdS energy	$E = \frac{1}{2}(a_i \,.\, a^i + b^r \,.\, b_r)$

Oscillator methods

[Günaydin-Marcus, Günaydin-Warner ’85, Günaydin-Minic-Zagermann ’98,…]

P = 1	Short/Singleton/ Doubleton	$D(j_1+1, j_1,0) \oplus D(j_2+1,0, j_2)$
P = 2	Semi-short	$D(j_1+j_2+2, j_1, j_2)$
P = 2	Chiral semi-short	$D(j_1+2+n, j_1,0) \oplus D(j_2+2+n,0, j_2)$
P > 2	Massive	$D(E, j_1, j_2)$

Similar results for SO(6,2)

Why do we care about short representations?

Eastwood-Vasiliev HS algebras

[Eastwood '02]

$$\mathrm{hs(d,2)} = \mathscr{U}(\mathrm{so(d,2)})/\mathscr{J}(\mathrm{so(d,2)})$$

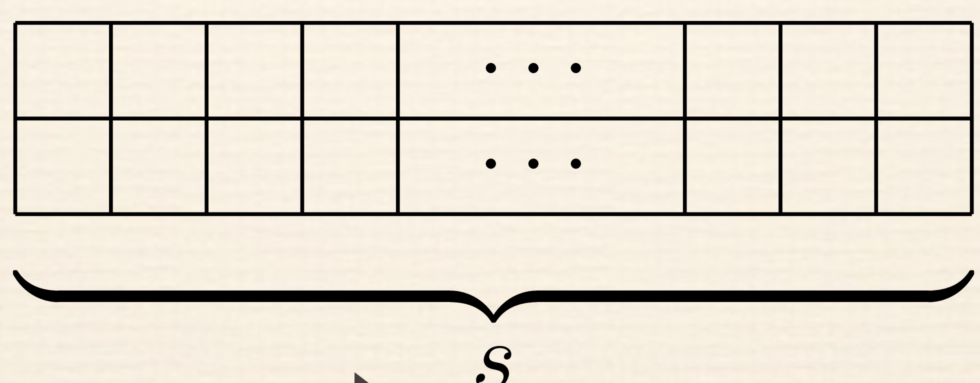
Universal enveloping algebra

Annihilator of scalar singleton/doubleton (Joseph ideal)

$$\Downarrow$$

$$\mathrm{hs(d,2)} = \mathscr{U}(\mathrm{so(d,2)}_{\text{scalar singleton/doubleton module}})$$

$$hs(d,2) \cong \bigoplus_{s=0}^{\infty}$$



trace-free part

Conformal Killing tensors

Minimal unitary irreducible representation (minrep)

Minrep: A unitary realization of a semi-simple Lie algebra on a Hilbert space of functions with minimal number of variables possible. [Joseph '74]

For SO(d,2) the scalar singleton/doubleton module is the minrep

d = 3 [Dirac '63, Flato-Fronsdal '78]

d = 4, 6 [Günaydin-Fernando '09-'10]

Oscillator representation (one pair) for SO(3,2) directly yields the scalar singleton or the minrep

Enveloping algebra $\downarrow$ [Vasiliev…, Günaydin ’89]

AdS_4 Higher spin (scalar/spinor) algebra hs(3,2)

Oscillator representation (two pairs) for SO(4,2) and SO(6,2) decomposes into infinite irreps (doubletons) including minrep

[Günaydin-Marcus, Günaydin-Nieuwenhuizen-Warner ’85]

Non-trivial constraints $\downarrow$ [Sezgin-Sundell ’01, Vasiliev ’05]

$AdS_{5/7}$ Higher spin (scalar/spinor) algebra hs(4,2), hs(6,2)

Generalized spacetimes & Quasiconformal Realizations (QCR)

[Günaydin-Koepsell-Nicolai '00; Günaydin and collaborators…]

Except for G_2, F_4 & E_8, certain non-compact real forms of all simple groups arise as conformal groups of formally real Jordan algebras

Simple Freudenthal triple systems (FTS) $\Leftrightarrow$ All simple Lie algebras (except Sl(2))

$$g = \underbrace{g^{-2}}_{\text{One-dimensional}} \oplus g^{-1} \oplus g^{0} \oplus g^{+1} \oplus \underbrace{g^{+2}}_{\text{One-dimensional}}$$

Using FTS triple product, define a quartic norm

Quasiconformal groups act geometrically on the space coordinatized by FTS and a singlet coordinate defined by the symplectic invariant of FTS

Invariance groups of “quartic light cones”

Geometric QCG action of $SU(2,2) \approx SO(4,2)$ in a 5-dimensional space

[Günaydin-Fernando ’09]

$\Big\downarrow$ Add a momenta for singlet & quantize

Minrep of $SU(2,2)$ in a 3 dimensional phase space

[KG-Günaydin ’13]

$\Big\downarrow$ Enveloping algebra

HS algebra!

Act I

Description of hs(4,2) using QCG

5-grading

[Günaydin-Pavlyk '06; Günaydin-Fernando '09]

$$so(4,2) = g^{-2} \oplus g^{-1} \oplus g^{0} \oplus g^{+1} \oplus g^{+2}$$

$$= \mathbf{1}^{-2} \oplus (\mathbf{2},\mathbf{2})^{-1} \oplus (\mathcal{D} \oplus sp(2,R) \oplus SO(2))^{0} \oplus (\mathbf{2},\mathbf{2})^{+1} \oplus \mathbf{1}^{+2}$$

Realized as bilinears of ordinary bosonic oscillators

Non-linearly realized using quartic invariant

Compact 3-grading

$$so(4,2) = (\text{Di-annihilation})^{-1} \oplus (su(2)_L \oplus su(2)_R \oplus E)^{0} \oplus (\text{Di-creation})^{+1}$$

Conformal (non-compact) 3-grading

$$so(4,2) = (P_\mu)^{-1} \oplus (so(3,1) \oplus so(1,1)_{\mathscr{D}})^{0} \oplus (K_\mu)^{+1}$$

Non-linear twistors

[KG-Günaydin '13]

$$[x\,,\,p] = i \qquad [d\,,\,d^\dagger] = 1 \qquad [g\,,\,g^\dagger] = 1$$

$Z_1 = \tfrac{1}{2}(x + ip - \mathscr{L}/x) - i\,g^\dagger$	$Y^1 = \tfrac{1}{2}(x - ip - \mathscr{L}/x) - i\,g$
$\hat{Z}_1 = \tfrac{1}{2}(x - ip - \mathscr{L}/x) + i\,g$	$\hat{Y}^1 = \tfrac{1}{2}(x + ip - \mathscr{L}/x) + i\,g^\dagger$
$Z_2 = \tfrac{1}{2}(x - ip + \mathscr{L}/x) - i\,d$	$Y^2 = \tfrac{1}{2}(x + ip + \mathscr{L}/x) + i\,d^\dagger$
$\hat{Z}_2 = \tfrac{1}{2}(x + ip + \mathscr{L}/x) + i\,d^\dagger$	$\hat{Y}^2 = \tfrac{1}{2}(x - ip - \mathscr{L}/x) - i\,d$

$$[X_\alpha\,,\,X_\beta\,] \propto {}^1\!/x\,(\delta_{\alpha\beta} \pm X_\alpha \pm X_\beta)$$

Generic Z or Y

$$\mathscr{L} = d^\dagger d - g^\dagger g - \tfrac{1}{2} + \zeta$$

Helicity $\zeta \in \mathbb{R}$

Generators of SO(4,2)

Translations

$$P_{\alpha\dot{\beta}}(\zeta) = (\sigma^{\mu} P_{\mu})_{\alpha\dot{\beta}} = - Z_{\alpha} \hat{Z}_{\dot{\beta}}$$

Special conformal

$$K^{\dot{\alpha}\beta}(\zeta) = (\bar{\sigma}^{\mu} K_{\mu})^{\dot{\alpha}\beta} = - \hat{Y}^{\dot{\alpha}} Y^{\beta}$$

Dilatation $\quad \mathscr{D}(\zeta) = (i/4)\left(Z_{\alpha} Y^{\alpha} + \hat{Y}^{\dot{\alpha}} \hat{Z}_{\dot{\alpha}}\right)$

Sl(2,C)

$$M_{\alpha}{}^{\beta}(\zeta) = (1/2)\left(Z_{\alpha} Y^{\beta} - {}^1\!/\!_2\, \delta_{\alpha}{}^{\beta} Z_{\gamma} Y^{\gamma}\right)$$

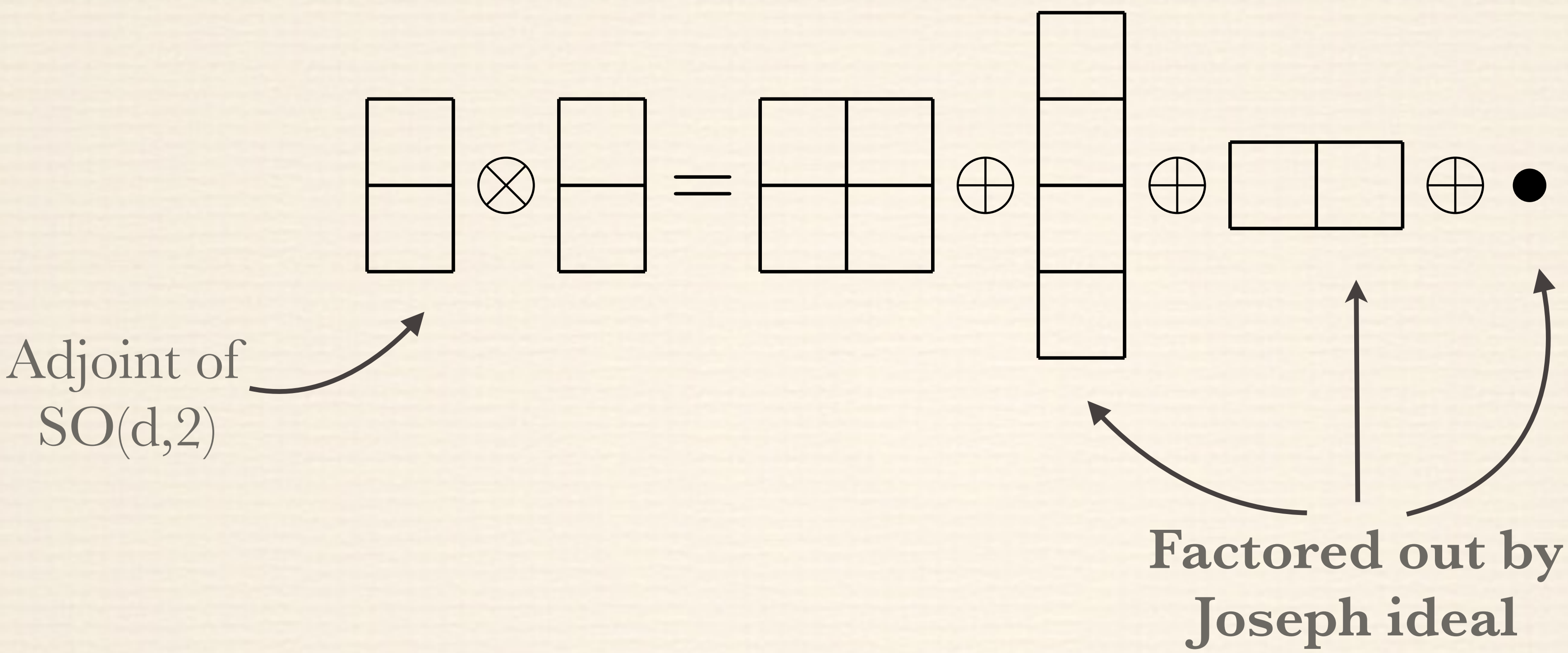
$$M^{\dot{\alpha}}{}_{\dot{\beta}}(\zeta) = -(1/2)\left(\hat{Y}^{\dot{\alpha}} \hat{Z}_{\dot{\beta}} - {}^1\!/\!_2\, \delta^{\dot{\alpha}}{}_{\dot{\beta}} \hat{Y}^{\dot{\gamma}} \hat{Z}_{\dot{\gamma}}\right)$$

Even though the generators are bilinears, Z & Y themselves are non-linear and it is an interacting realization

$\alpha, \beta = 1,2 \qquad \dot{\alpha}, \dot{\beta} = 1,2$

Joseph ideal

$U(so(4,2)) = \sum$ Symmetric tensor products in adjoint



$$J_{ABCD} = 1/2\ \{M_{AB}, M_{CD}\} - M_{AB}\odot M_{CD} - 1/60\ \langle M_{AB}, M_{CD}\rangle$$

Generators J_{ABCD} vanish identically in minrep

Joseph ideal

$P^2 = P^\mu \cdot P_\mu = 0$ ← Massless → $K^2 = K^\mu \cdot K_\mu = 0$

$\frac{1}{2}\, \varepsilon^{\mu\nu\rho\sigma} P_\nu \cdot M_{\rho\sigma} = 0$ ← Pauli - Lübanski vector → $\frac{1}{2}\, \varepsilon^{\mu\nu\rho\sigma} K_\nu \cdot M_{\rho\sigma} = 0$

$P^\mu \cdot (M_{\mu\nu} + \eta_{\mu\nu}\, \Delta) = 0$ $(M_{\mu\nu} + \eta_{\mu\nu}\, \Delta) \cdot K^\mu = 0$

Fixes Casimir to a c-number ⟶ $4\, \Delta \cdot \Delta + M^{\mu\nu} \cdot M_{\mu\nu} + P^\mu \cdot K_\mu = 0$

$$\eta^{\mu\nu} M_{\mu\rho} \cdot M_{\nu\sigma} - P_{(\rho} \cdot K_{\sigma)} + 2\eta^{\rho\sigma} = 0$$

$$M_{\mu\nu} \cdot M_{\rho\sigma} + M_{\mu\sigma} \cdot M_{\nu\rho} + M_{\mu\rho} \cdot M_{\sigma\nu} = 0$$

$$\Delta \cdot M_{\mu\nu} + P_{[\mu} \cdot K_{\nu]} = 0$$

$$\mu, \nu = 0,1,\ldots,3$$

Deformed Joseph ideal

$P^2 = P^\mu \cdot P_\mu = 0$ ← Massless → $K^2 = K^\mu \cdot K_\mu = 0$

$\frac{1}{2}\, \varepsilon^{\mu\nu\rho\sigma} P_\nu \cdot M_{\rho\sigma} = \zeta\, P^\mu$ ← Pauli - Lübanski vector → $\frac{1}{2}\, \varepsilon^{\mu\nu\rho\sigma} K_\nu \cdot M_{\rho\sigma} = -\, \zeta\, K^\mu$

$P^\mu \cdot (M_{\mu\nu} + \eta_{\mu\nu}\, \Delta) = 0$ $\qquad$ $(M_{\mu\nu} + \eta_{\mu\nu}\, \Delta) \cdot K^\mu = 0$

Fixes Casimir to a c-number ⟶ $4\, \Delta \cdot \Delta + M^{\mu\nu} \cdot M_{\mu\nu} + P^\mu \cdot K_\mu = 0$

$$\eta^{\mu\nu} M_{\mu\rho} \cdot M_{\nu\sigma} - P_{(\rho} \cdot K_{\sigma)} + 2\eta_{\rho\sigma} = (\zeta^2/\, 2)\, \eta_{\rho\sigma}$$

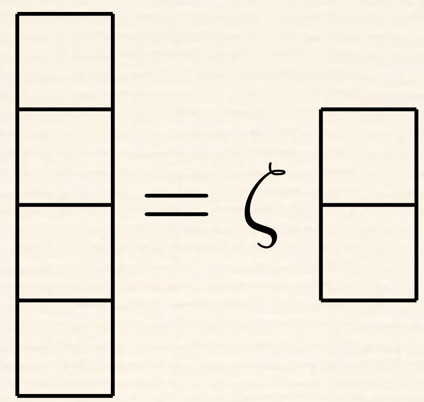
$$M_{\mu\nu} \cdot M_{\rho\sigma} + M_{\mu\sigma} \cdot M_{\nu\rho} + M_{\mu\rho} \cdot M_{\sigma\nu} = \zeta\, \varepsilon_{\mu\nu\rho\sigma}\, \Delta$$

$$\Delta \cdot M_{\mu\nu} + P_{[\mu} \cdot K_{\nu]} = -(\zeta/\, 2)\, \varepsilon_{\mu\nu\rho\sigma}\, M^{\rho\sigma}$$

$\mu, \nu = 0,1,\ldots,3$

Role of ζ in deformed HS algebras

For $\zeta \neq 0$



Thus even though, 4-row diagrams do not vanish, they can be dualized to two row diagrams and the deformed HS algebras are still Vasiliev type algebras.

A one-parameter family of HS algebras in 4d were also found by Young Tableaux analysis [Boulanger-Skvortsov '11]

Supersymmetric extension SU(2,2|N)

Fermionic oscillators $\{\xi_I , \xi^J\} = \delta_I{}^J$ (I , J=1,…,N)

Odd generators

$$Q^I{}_\alpha = Z^s_\alpha(\zeta)\xi^I, \qquad \bar{Q}_{I\dot{\alpha}} = -\xi_I \widetilde{Z}^s_{\dot{\alpha}}(\zeta)$$

$$S_I{}^\alpha = -\xi_I Y^{s\alpha}(\zeta), \qquad \bar{S}^{I\dot{\alpha}} = \widetilde{Y}^{s\dot{\alpha}}(\zeta)\xi^I$$

SU(N) R-symmetry generators

$$R^I{}_J = \xi^I \xi_J - \frac{1}{N}\delta^I{}_J \xi^K \xi_K$$

Central charge

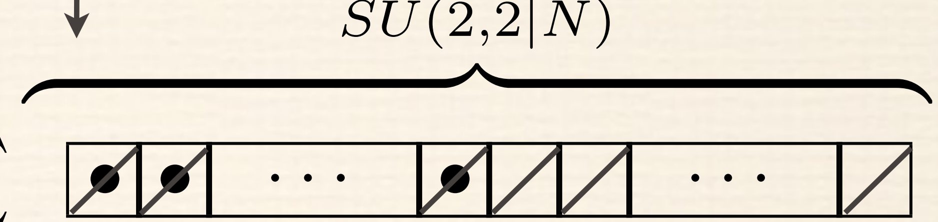
$$\mathcal{L}_\zeta \longrightarrow \mathcal{L}^s_\zeta = N_d - N_g + N_\xi + \zeta - \frac{5}{2}$$

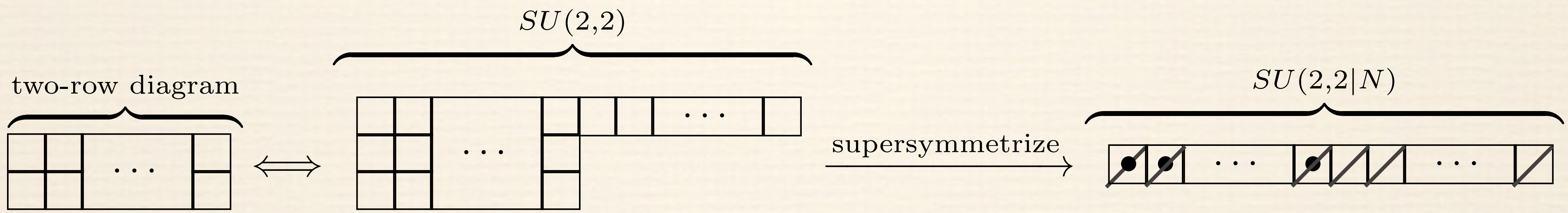
Supersymmetric extensions

Superconformal group in 4d - SU(2,2|N)

Enveloping algebra

$$HS[SU(2,2|N);0] = \sum_{\oplus}$$





Maximal finite dimensional subalgebra is SU(2,2|N) and HS[SU(2,2|N);ζ] contains HS algebras of various irreps in supermultiplet of SU(2,2|N) as subalgebras

QCG for SO(3,2) ≈ Sp(4,R)

Quartic invariant (I_4)

$\downarrow$

Non linearities in QCG

Symplectic groups ⇒ I_4 vanishes ⇒ QCG reduces to usual bilinears

Fock space of oscillators decomposes into 2 irreps of SO(3,2) namely scalar and spinor singletons

Act II

Description of hs(6,2) using QCG

5-grading

[Günaydin-Pavlyk '06; Günaydin-Fernando '09-'10]

$$so(6,2) = g^{-2} \oplus g^{-1} \oplus g^{0} \oplus g^{+1} \oplus g^{+2}$$

$$= \mathbf{1}^{-2} \oplus (\mathbf{4},\mathbf{2})^{-1} \oplus (\mathcal{D} \oplus SO(4) \oplus sp(2,R))^{0} \oplus (\mathbf{4},\mathbf{2})^{+1} \oplus \mathbf{1}^{+2}$$

Realized as bilinears of ordinary bosonic oscillators

Non-linearly realized

Compact 3-grading

$$so(6,2) = (\text{Di-annihilation})^{-1} \oplus (so(6) \oplus E)^{0} \oplus (\text{Di-creation})^{+1}$$

Conformal (non-compact) 3-grading

$$so(6,2) = (P_\mu)^{-1} \oplus (so(5,1) \oplus so(1,1)_{\mathscr{D}})^{0} \oplus (K_\mu)^{+1}$$

Massless representations in 6d

	SO(4,2)	SO(6,2)
Little group of massless particles	U(1)	$SO(4) = SU(2)_L \times SU(2)_A$
Labels	Continuous ζ	Discrete (j_L , j_A)
Non-linear twistors	$\mathscr{L} = d^\dagger d - g^\dagger g - \frac{1}{2} + \zeta$	$T_\pm, T_0 : SU(2)_T$ generators

Conformally massless reps are of form $j_L\, j_A = 0$ i.e. $(j_L , 0)$ or $(0 , j_A)$

"Orbital" generators of $SU(2)_L$ get extended to "total angular momentum" $SU(2)_T$ by adding "spin" generators $SU(2)_S$

$$T_i = L_i + S_i$$

Generators of SO(6,2)

[KG-Günaydin '14]

Translations

$$P_{\alpha\beta} = (\Sigma^{\mu}\, P_{\mu})_{\alpha\beta} = Z_{\alpha}{}^{i}\, \hat{Z}_{\beta}{}^{j}\, \varepsilon_{ij}$$

Special conformal

$$K^{\alpha\beta} = (\, \bar{\Sigma}^{\mu}\, K_{\mu})^{\alpha\beta} = Y^{\alpha i}\, \hat{Y}^{\beta j}\, \varepsilon_{ij}$$

Dilatation

$$\mathscr{D} = ({}^{i}\!/_{8})\, (Z_{\alpha}{}^{i}\, Y^{\alpha j} - Y^{\alpha i}\, \hat{Z}_{\alpha}{}^{j})\, \varepsilon_{ij}$$

SO(5,1)

$$M_{\alpha}{}^{\beta} = ({}^{1}\!/_{2})\, (Y^{\beta i}\, \hat{Z}_{\alpha}{}^{j} - {}^{1}\!/_{4}\, \delta_{\alpha}{}^{\beta}\, Y^{\gamma i}\, \hat{Z}_{\gamma}{}^{j}\,)\, \varepsilon_{ij}$$

$$= -({}^{1}\!/_{2})\, (Z_{\alpha}{}^{i}\, \hat{Y}^{\beta j} - {}^{1}\!/_{4}\, \delta_{\alpha}{}^{\beta}\, Z_{\gamma}{}^{i}\, \hat{Y}^{\gamma j})\, \varepsilon_{ij}$$

$\alpha\,,\, \beta = 1,2,3,4 \qquad i\,,\, j = 1,2$

Joseph ideal

$$P^2 = P^\mu \cdot P_\mu = 0 \quad \leftarrow \quad \text{Massless} \quad \rightarrow \quad K^2 = K^\mu \cdot K_\mu = 0$$

Analogs of Pauli - Lübanski vector

$$A_{\nu\rho\sigma} = P_{[\nu} \cdot M_{\rho\sigma]} = 0 \quad \leftarrow \qquad \rightarrow \quad E_{\nu\rho\sigma} = K_{[\nu} \cdot M_{\rho\sigma]} = 0$$

$$P^\mu \cdot (M_{\mu\nu} + \eta_{\mu\nu}\,\Delta) = 0 \qquad (M_{\mu\nu} + \eta_{\mu\nu}\,\Delta) \cdot K^\mu = 0$$

Fixes Casimir to a c-number $\longrightarrow$ $6\,\Delta \cdot \Delta + M^{\mu\nu} \cdot M_{\mu\nu} + 2\,P^\mu \cdot K_\mu = 0$

$$\eta^{\mu\nu} M_{\mu\rho} \cdot M_{\nu\sigma} - P_{(\rho} \cdot K_{\sigma)} + 4\eta^{\rho\sigma} = 0$$

$$M_{\mu\nu} \cdot M_{\rho\sigma} + M_{\mu\sigma} \cdot M_{\nu\rho} + M_{\mu\rho} \cdot M_{\sigma\nu} = 0$$

$$\Delta \cdot M_{\mu\nu} + P_{[\mu} \cdot K_{\nu]} = 0$$

$$\mu, \nu = 0,1,\ldots,5$$

Deformed Joseph ideal

$P^2 = P^\mu \cdot P_\mu = 0$ ← Massless → $K^2 = K^\mu \cdot K_\mu = 0$

$A_{\nu\rho\sigma} = \tilde{A}_{\nu\rho\sigma}$ ← Self dual and anti-self-dual → $E_{\nu\rho\sigma} = - \tilde{E}_{\nu\rho\sigma}$

$P^\mu \cdot (M_{\mu\nu} + \eta_{\mu\nu}\,\Delta) = 0$ $\qquad$ $(M_{\mu\nu} + \eta_{\mu\nu}\,\Delta) \cdot K^\mu = 0$

Fixes Casimir to a c-number ⟶ $6\,\Delta \cdot \Delta + M^{\mu\nu} \cdot M_{\mu\nu} + 2\,P^\mu \cdot K_\mu = 0$

$$\eta^{\mu\nu} M_{\mu\rho} \cdot M_{\nu\sigma} - P_{(\rho} \cdot K_{\sigma)} + 2\eta_{\rho\sigma} = 2t(t+1)\,\eta_{\rho\sigma}$$

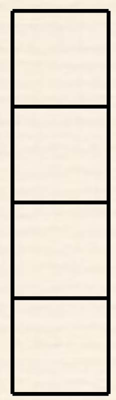
deformation SU(2) spin

$$M_{\mu\nu} \cdot M_{\rho\sigma} + M_{\mu\sigma} \cdot M_{\nu\rho} + M_{\mu\rho} \cdot M_{\sigma\nu}$$
$$\|$$
$$\varepsilon_{\mu\nu\rho\sigma}{}^{\delta\tau}\,(\Delta \cdot M_{\delta\tau} + P_{[\delta} \cdot K_{\tau]})$$

$\mu, \nu = 0,1,\ldots,5$

Deformed AdS_7/CFT_6 HS algebra

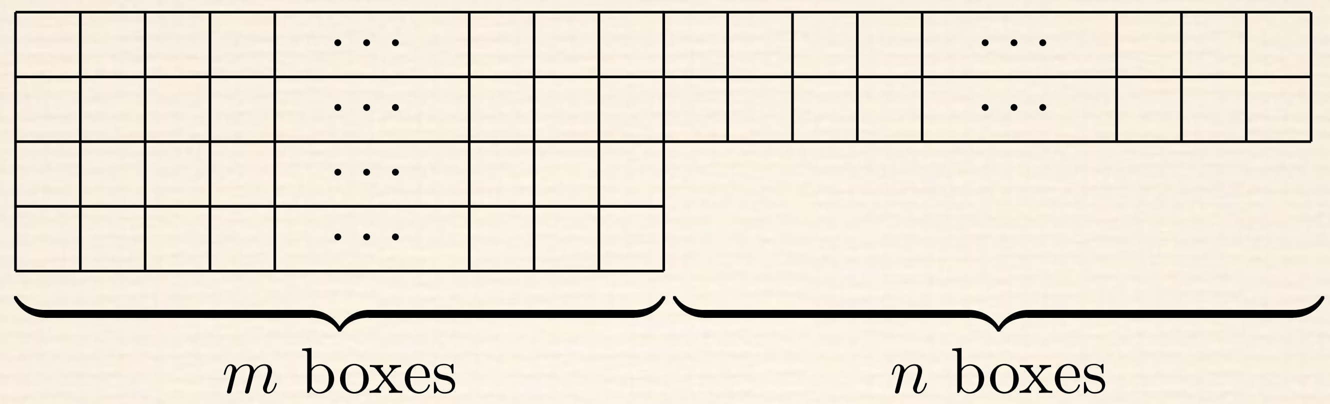
$$hs(6,2;t) = \mathscr{U}\,(so(6,2)_{QCR})$$

However  does not vanish for $t \neq 0$, but, it satisfies an 8-dimensional self duality condition $\Leftrightarrow$ 3-form gauge field with a self dual field strength

AdS_7: 3-form gauge fields satisfying odd dimensional self duality

6d: Conformal 2-form fields with a self dual field strength (tensor field of (2,0) supermultiplet)

hs(6,2;t) generators include



This suggests theories based on discrete deformations of the minrep describe HS theories of Fradkin-Vasiliev type in AdS_7 coupled to tensor fields that satisfy self-duality conditions and their higher extensions

Act III

HS holographic dualities and correlation functions

HS holography in AdS_d

What is known?

Vasiliev has constructed d-dimensional HS theory of interacting HS fields

[Vasiliev '11 (cubic coupling in AdS_d)]

Totally symmetric massless HS fields

But there are mixed symmetry massless fields in $d>4$

Symmetry algebras for these mixed symmetry fields for AdS_5 and AdS_7 [KG-Günaydin '13, '14]

AdS theories

[Metsaev '95, Alkalaev-Shaynkman-Vasiliev '03, Alkalaev '12]

???

Expected spectrum of mixed symmetry massless fields in AdS$_5$

Symmetric boundary theory (for $j_1 = j_2$) [Heidenreich '80]

$$D\left(j_1+1, j_1, 0\right) \otimes D\left(j_2+1, 0, j_2\right) = \sum_{s=0}^{\infty} D\left(j_1+j_2+2+s, j_1+\frac{s}{2}, j_2+\frac{s}{2}\right)$$

Chiral boundary theory

$$\begin{aligned} D\left(j_1+1, j_1, 0\right) \otimes D\left(j_2+1, j_2, 0\right) \quad &= \quad \sum_{s=|j_1-j_2|}^{j_1+j_2} D\left(j_1+j_2+2, s, 0\right) \\ &\oplus \sum_{s=0}^{\infty} D\left(j_1+j_2+2+s, j_1+j_2+\frac{s}{2}, \frac{s}{2}\right) \end{aligned}$$

Tensoring in QCG will lead to an interacting realization of massless fields in bulk for arbitrary ζ [KG-Günaydin (work in progress)]

Spin one boundary theory, one loop corrections to vacuum energy and 4d anomaly coefficients recently studied by [Beccaria-Tseytlin '14]

To be free or to interact, that is the question...

Under certain reasonable assumptions (unitarity, finite & unique stress tensor...), the dual CFT in *3 dimensions* with exactly conserved HS symmetry is free!

[Maldacena-Zhiboedov '11, Boulanger-Ponomarev-Skvortsov-Taronna '13]

There are certainly free CFTs dual to HS theories in $d > 3$

[Giombi-Klebanov-Tseytlin '14]

No general theorem that forbids interactions in $d > 3$, but no known examples of an interacting theory with exactly conserved HS symmetry either

QCG is non-linear

Generators of conformal transformations contain terms that are cubic and quartic in oscillators

$\Downarrow$

Interacting realization!

However for $\zeta \in \mathbb{Z}$, isomorphic to doubletons which are free field realizations (bilinears)

What does it mean for the CFTs whose conserved charges are given by QCG generators?

Interacting example in one dimension

D(2,1;α) superconformal quantum mechanics, interacting matrix model with non-linear supersymmetry transformations

[Fedoruk-Ivanov-Lechtenfeld ‘09]

1-1 mapping of quantum generators of superconformal symmetry with QCG generators for deformed minreps of D(2,1;α)

[KG-Günaydin ‘12]

Probing CFTs: Correlation functions

Conformal symmetry constrains correlation functions but HS symmetries constrain them even further

[Maldacena-Zhiboedov '11]

Weakly broken HS symmetry for even low N

3d Ising Model: $\tau_4 \approx 1.0208(12)$ [Campostrini et al '97]

$1.02 \leq \tau_s \leq 1.037$, $s \geq 6$ [Komargodski-Zhiboedov '13]

Standard oscillator approach: Need to impose constraints/projectors to factor out the ideal

QCG approach: no need for projectors & straightforward supersymmetric extensions

(Part of) Vasiliev's equations

(relevant for bulk to boundary propagator)

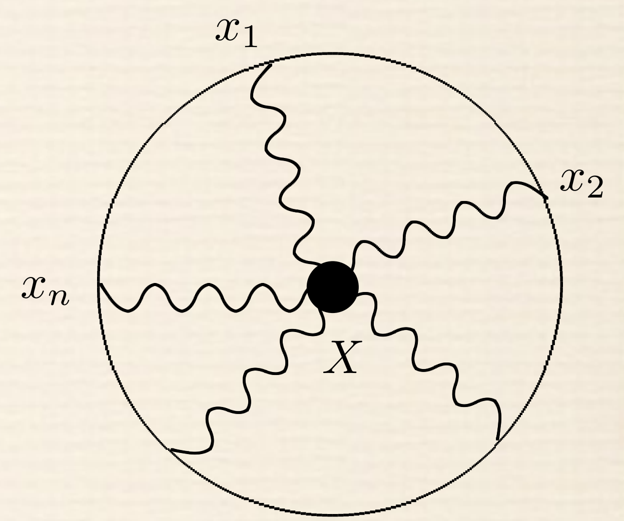
$$d\Omega + \Omega \star \Omega = 0$$

$$dB + \Omega \star B + B \star \bar{\Omega} = 0$$

Twisted adjoint representation

Intertwines HS Master field containing HS field strengths in the bulk and the generating function of conserved currents on the boundary

Flat connection that defines AdS background



$$< j\,(x_1) \ldots j\,(x_n) > \; = \sum \mathrm{Tr}\,(\Phi\,(X, x_1) \star \ldots \star \Phi\,(X, x_n))$$

n-point correlation function

Depends on B and transforms in adjoint

X dependence drops out because change in X is a HS gauge transformation

[Columbo, Sundell '12; Didenko, Skvortsov '12]

3d: $< j_{s1}\; j_{s2}\; j_{s3} > =$ Free Boson + Free fermion

[Maldacena-Zhiboedov '11]

Light cone limit only works for symmetric currents

4d: $< j_{s1}\; j_{s2}\; j_{s3} > =$ Free Boson + Free fermion + Free spin-1

[Alba-Diab '13,Boulanger-Ponomarev-Skvortsov-Taronna '13]

Totally symmetric currents

Compute the HS correlators for mixed symmetry fields based on deformed hs(4,2; ζ) using Vasiliev equations for bulk-boundary propagator

[KG-Günaydin-Skvortsov-Taronna (work in progress)]

Encore

QCG methods provide a natural and unified framework for studying HS (super) symmetries in four & six dimensions and computing n-point correlation functions

Richer spectrum of mixed symmetry massless fields in AdS_d ($d > 4$) leads to a variety of CFTs and new HS holographic dualities

Breaking HS symmetries weakly can lead to interesting non-trivial CFTs (with good control) in 4d and 6d

Things not addressed

Breaking HS symmetry in higher dimensions?

Application of non-linear twistors to spin chain models associated with $\mathscr{N} = 4$ SYM

(Spectral parameter or unphysical helicities for scattering amplitudes in $\mathscr{N} = 4$ SYM corresponds to deformation parameter ζ)

[Ferro-Lukowski-Meneghelli-Plefka-Staudacher ’12, ’13]

Thank you for your attention